Product Differentiation in the Presence of Positive and Negative Network Effects*

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Abstract

Using two standard location models, we investigate price competition and divergence from optimal product differentiation when consumer preferences are influenced by the number of consumers purchasing the same brand or shopping at the same store. Negative network effects tend to lessen competition and increase prices whereas positive network effects (bandwagon effects) make competition fiercer and lead to lower prices. Furthermore, in the duopoly case, an increase in the total population may hurt the clients of a store despite the fact that they benefit from price cuts. Finally, under free entry, increasing the population may lead to a decrease in the equilibrium number of stores and always enlarges the divergence between the equilibrium and optimal numbers of stores.

Keywords: Product Differentiation, Location, Network Effects, Network Externalities.

JEL Classification Numbers: L1, R3

Draft (internal) = nst51.tex

Suggested Citation: Center for Economic Policy Research (CEPR), Discussion Paper No. 1306, December 1995. Web site: `www.cepr.org/pubs/dps/DP1306.asp`

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*We thank Olivier Jeanne for a very useful suggestion. The third author thanks PIR-VILLE (CNRS) for financial support.

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1 Introduction

Ever since Veblen [1899], it is a well-documented fact that consumer choices are not only based upon their own preferences and income; they are also affected by the consumption choice of others. Such influences have proven to be important in many markets where the decision to buy from a particular vendor is positively or negatively affected by the number of consumers purchasing the same brand or patronizing the same store. The corresponding effects are known as bandwagon, congestion, or snob/conformity effects (Leibenstein [1950]). The microeconomic foundations of such effects as well as the market and welfare implications of this type of consumer behaviour were not fully explored. Later, they were characterized by the assumption that preferences exhibit network externalities (Katz and Shapiro [1985]).

There is a whole body of literature addressing *network goods* where consumers' preferences depend directly upon the clientele size (see Farrell and Saloner [1987] and David and Greenstein [1990] for recent surveys). Typically, the willingness to pay for such a good increases with the number of customers who buy it because the good becomes more useful when the number of other consumers connected to the network rises (e.g., telephone, fax, e-mail). In the same vein, Becker [1991] has analyzed a provocative example in which one restaurant eventually captures almost the entire business while its competitor has a negligible market share in a market where no restaurant has an ex ante technological advantage. Becker suggests that a positive consumption externality, that is, the demand for a store is positively related to its number of customers, may explain why similar stores experience vastly different sales patterns over long time periods (see also Rohlfs [1974] for the construction of such a demand in the case of a communication service). Recently, Karni and Levine [1994] have provided some game-theoretic foundations for such an externality (see Kirman [1993] for a different approach). However, the choice of specific goods in certain social contexts may also be associated with bandwagon or snob effects. In this respect, the work of Frank [1985] has provided new and interesting insights, and has triggered a new flow of research about the role of conformity and snobbism.

Finally, there are other reasons explaining why the number of customers affects the relative attractiveness of a particular brand, store or vendor. For example, Chou and Shy [1990, 1995] as well as Church and Gandal [1992] have shown that, even without assuming network externalities, the existence of preferences for complementary supporting services (such as software) may generate consumer behavior similar to the behavior of consumers whose preferences exhibit network externalities. Similarly, stores with a large number of consumers tend to provide more services to its clients (after sales service and large parking lots) that make their stores more attractive.[1] In contrast, crowded stores may deter consumers who refuse to incur the corresponding congestion costs and prefer to shop at other places just because they hate congestion and value quietness, thus exhibiting a behavior consistent with negative network effects.

Somewhat surprisingly, spatial competition and retailing models have disregarded network influences on consumer behavior by assuming that people always buy from the cheapest store (see Gabszewicz and Thisse [1992] for a recent survey). This also holds for product differentiation theory which has neglected to account for these phenomena. Instead, recent developments have focused on the role of observable and unobservable attributes in the formation of idiosyncratic tastes and on their impact on market equilibrium (Anderson, de Palma, and Thisse [1992]). Still, casual observations suggest that network externalities are present and may well explain the failure or the fast penetration of some products and services as well as the segmentation of some markets. Conversely, the literature on consumption externalities has neglected the fact that stores are not necessarily clustered and the consumers face transportation costs in addition to the consumption externality when choosing which brand to buy or which store to visit. By explicitly integrating the location dimension into these environments, our model becomes general enough to capture both positive and negative network effects and to contrast stores' strategies under these two extreme consumer behavior.

In this paper, we propose to graft the consumption (or network) externality model onto the location models in order to highlight the role of the network externalities in retailing and

[1] Note, however, that the relevance of some of those effects have been criticized by Liebowitz and Margolis [1994]

spatial competition. We then investigate the impact of stores' location on equilibrium prices, market shares, and welfare. It is worth noting that transportation costs (or consumers' disutility) substantially differ from consumption externalities. Consumers' transportation costs are independent of the number of consumers purchasing from the same store, whereas positive and negative network effects are external to consumers. For this reason, it is important to develop a model where both transportation costs *and* network effects are mutually present in the same model. We emphasize this point since bandwagon effects are sometimes viewed as "negative" transportation costs since they tend to intensify price competition among stores. However, as demonstrated in this paper, they are intrinsically different since network effects affect all buyers whereas transportation costs do not confer any externality.

The analysis developed below shows that integrating network effects into product differentiation models may lead to very different outcomes.[2] This is best shown by the following results. When positive network externalities are present, price competition is fiercer and results in low equilibrium prices; however, both stores remain in business provided that transportation costs are not "too low". When the network externalities are strong enough, they dominate the transportation cost effect, resulting in a single store serving the entire market (as in Becker's restaurant example). Hence, like in contestable markets, the active store prices below the monopoly price because of entry threats.

At the other extreme, when consumers' preferences exhibit negative network externalities, price competition is relaxed and firms have more market power. Hence, the market is characterized by higher equilibrium prices. This in turn may invite more stores to enter so that negative network externalities induce a larger variety of brands as often observed in luxury good industries.

Finally, the integration network externalities into location models allows us to investigate the effects of changing the population size on (a) the clientele size of each store and (b) the welfare of existing consumers. We demonstrate that while under positive network externalities, utility of existing consumers generally rise with an increase in consumer pop-

[2] Note that de Palma and Leruth [1993] have studied a network externality model where product differentiation is expressed through the logit. However, they concentrated on symmetric solutions only.

ulation, under negative network externalities consumers' welfare decreases with an increase in population. In addition, we show that under negative network externalities, an increase in population may *reduce* a store's clientele size.

The paper is organized as follows. In Section 2 we consider the classical Hotelling model where two stores compete for customers distributed along Main street, and analyze the effects of population on prices, market shares, profits, and consumer welfare when positive or negative network externalities are present. Section 3 introduces positive and negative network externalities into the circular city model, and analyzes product diversity and divergence from optimum diversity, as in Dixit and Stiglitz [1977] and Salop [1979]. Section 4 concludes.

2 The Hotelling Linear City Model

2.1 The Model

Consider the Hotelling [1929] model where two stores selling a homogeneous product are located on the interval $[0, L]$ with $L > 0$. Store A locates $a \geq 0$ units of distance from point 0 and store B locates $b \geq 0$ units of distance from point L, where $a + b \leq L$. Without loss of generality, we assume that $a \geq b$; we will refer to store A as the *large* store and to store B as the *small* one. We suppose that production is costless and denote by p_i the mill price charged by store i, $i = A, B$. Figure 1 illustrates the location of the stores.

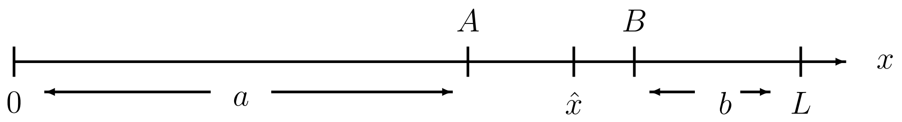

Figure 1: Hotelling's linear city model

There is a continuum of N consumers uniformly distributed on the interval $[0, L]$. A consumer located x units of distance from store i 'pays' a transportation cost of τx for buying at i, where $\tau \geq 0$ measures the per unit of distance transportation cost (or distaste's cost of buying away from his ideal brand).

The present model differs from the traditional Hotelling model in the following aspect. Let n_A denote the number of consumers buying from store A and n_B the number of consumers buying from B. Assuming that each consumer buys one unit of the product yields $n_A + n_B = N$. We define the utility of consumer x, $x \in [0, L]$ by

$$U_x \equiv \begin{cases} \alpha n_A - p_A - \tau|x - a| & \text{when buying from } A \\ \\ \alpha n_B - p_B - \tau|L - b - x| & \text{when buying from } B. \end{cases} \tag{1}$$

where α (which can be positive or negative) measures how the number of consumers purchasing from the same store is important to each one of them. Note that for $\alpha = 0$, the model reduces to the standard Hotelling model.

Definition 1 *Let $\beta \equiv \alpha N/L$. Consumer preferences are said to exhibit*

- **negative network effects** *if $\beta < 0$,*
- **weakly positive network (bandwagon) effects** *if $0 < \beta < \tau$,*
- **strong positive network (bandwagon) effects** *if $\beta > \tau$.*

The first part of Definition 1 corresponds to $\alpha < 0$ in the utility function (1) and reflects *negative network effects* (due to congestion or snob effects) since the utility of each consumer *decreases* with the number of other consumers purchasing the same brand or from the same store; for example disutility may arise from congestion as in Kohlberg [1983]. Noting that N/L is the consumers' density, the second part of Definition 1 means that the bandwagon effects (the gain to a consumer from the clientele size) is dominated by the loss from having to travel additional distance to the store. The opposite interpretation holds for the third part of the definition.

2.2 Equilibrium under negative or weak network effects

We first define an equilibrium where there exists a consumer denoted by $\hat{x}$ so that all consumers indexed by $x \in [0, \hat{x}]$ purchase from A and all consumers indexed by $x \in (\hat{x}, L]$ buy from B. In this case, $n_A = \hat{x}N/L$ and $n_B = (L - \hat{x})N/L$. Substituting into (1), the marginal consumer $\hat{x}$ must satisfy

$$\frac{\alpha N \hat{x}}{L} - p_A - \tau(\hat{x} - a) = \frac{\alpha(L - \hat{x})N}{L} - p_B - \tau(L - b - \hat{x}).$$

Setting $\beta = \alpha N/L$ and solving for $\hat{x}$ yields

$$\hat{x}(p_A, p_B) = \begin{cases} 0 & \text{if } p_A - p_B > \tau(L - b + a) - \beta L \equiv \phi_H \\ L & \text{if } p_A - p_B < -\tau(L + b - a) + \beta L \equiv \phi_L \\ \frac{p_B - p_A}{2(\tau - \beta)} + \frac{\tau(L - b + a)}{2(\tau - \beta)} - \frac{\beta L}{2(\tau - \beta)} & \text{if } \phi_L \leq p_A - p_B \leq \phi_H. \end{cases} \tag{2}$$

Definition 2 *An equilibrium is a pair* (p_A^h, p_B^h)*, such that, given* p_B^h*,* p_A^h *solves* $\max_{p_A} \pi_A \equiv p_A \hat{x}(p_A, p_B^h)$ *and, given* p_A^h*,* p_B^h *solves* $\max_{p_B} \pi_B \equiv p_B[L - \hat{x}(p_A^h, p_B)]$*; where* $\hat{x}(p_A, p_B)$ *is given in (2).*

It is readily verified that for an equilibrium to exist it must be that $\phi_L \leq p_A^h - p_B^h \leq \phi_H$. In this case, the best-response functions are:

$$p_A = R_A(p_B) = \frac{1}{2}[p_B + \tau(L - b + a) - \beta L] \text{ and } p_B = R_B(p_A) = \frac{1}{2}[p_A + \tau(L + b - a) - \beta L]. \tag{3}$$

They are drawn in Figure 2.

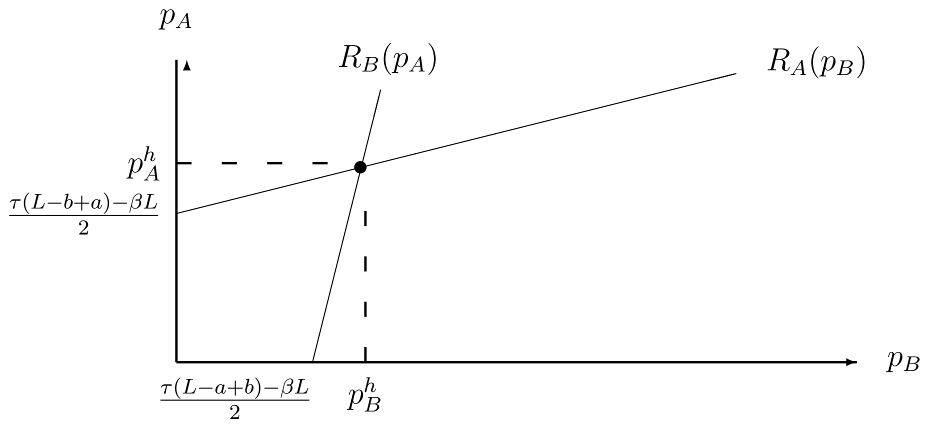

Figure 2: Negative and weakly positive network effects: best-response functions ($a > b$)

Assuming that an equilibrium exists, solving (3) yields

$$p_A^h = \frac{\tau(3L+a-b)}{3} - \beta L \quad \text{and} \quad p_B^h = \frac{\tau(3L-a+b)}{3} - \beta L. \tag{4}$$

Substituting into (2) yields the market share of each store:

$$\hat{x} = \frac{L}{2} + \frac{\tau(a-b)}{6(\tau-\beta)} \quad \text{and} \quad L - \hat{x} = \frac{L}{2} + \frac{\tau(b-a)}{6(\tau-\beta)}. \tag{5}$$

Recalling that $n_A = \hat{x}N/L$ and $n_B = (L-\hat{x})N/L$, the profit of each store is given by

$$\pi_A^h = p_A^h n_A = \frac{N[\tau(3L+a-b)-3\beta L]^2}{18L(\tau-\beta)} \quad \text{and} \quad \pi_B^h = p_B^h n_B = \frac{N[\tau(3L+b-a)-3\beta L]^2}{18L(\tau-\beta)}. \tag{6}$$

It is well known that, even in the absence of network effects, a pure strategy equilibrium does not exist in the Hotelling model when the stores are located 'close' to each other (see d'Aspremont, Gabszewicz and Thisse [1979]). Their analysis can easily be extended to the present case by identifying the restrictions on a and b that would ensure that neither store could increase its profit by undercutting its rival and serving the entire market. The conditions for existence are stated in the following proposition.

Proposition 1 *The necessary and sufficient conditions for a price equilibrium in pure strategies to exist are*

$$[\tau(3L+a-b)-3\beta L]^2 \geq 12L[(2b+a)\tau - 3b\beta](\tau-\beta) \tag{7}$$

$$[\tau(3L+b-a)-3\beta L]^2 \geq 12L[(2a+b)\tau - 3a\beta](\tau-\beta).$$

Furthermore, this equilibrium is unique and is given by (4) and (5).

Clearly, substituting $\beta = 0$ into (7) yields the conditions given in d'Aspremont et al [1979]. If both stores are located at the same distance from the edges, $a = b$, then the conditions (7) reduce to $a = b \leq L/4$.

Equation (3) and Figure 2 reveal that an increase in the parameter β shifts the best response functions inward. Hence,

Proposition 2

1. *Suppose that consumer preferences exhibit negative network effects. Then, both equilibrium prices p^h_A and p^h_B increase when*

   (a) *negative network effects become more significant in consumers' preferences (α becomes more negative);*

   (b) *there are more consumers per unit of area (N increases).*

2. *Suppose that consumers' preferences exhibit weakly positive network effects. Then, both equilibrium prices p^h_A and p^h_B are reduced when*

   (a) *network effects become more significant in consumers' preferences (α increases),*

   (b) *there are more consumers per unit of area (N increases).*

The first part of Proposition 2 states that negative network effects (whether due to congestion or pure snobbism) reduce competition between stores and, therefore, increase equilibrium prices. The reason is that when $\alpha < 0$, a price reduction by one store is less effective in increasing the store's market share since the increase in market share would reduce the utility of buyers and, therefore, their willingness to buy from this store. Hence, under negative network effect, stores would maintain a high price in order to restrict the number of consumers, thereby charging even a higher price, and so on.... Also, an increase population density will raise equilibrium prices under negative network effects because each store would attempt to further reduce its clientele size by charging a higher price.

In contrast, the existence of weakly positive network effects intensifies competition between the two stores. The intuition behind this result is as follows. When $\alpha > 0$, stores have more market share to gain when they cut their prices (compared to the case where $\alpha = 0$), since a reduction in price increases the market share of the undercutting store and, hence, further increases its market share because the corresponding store becomes even more attractive due to the increase in the number of consumers buying from it. More generally, an increase in β (resulting from either an increase in the preference for clientele size or an increase in population) makes competition fiercer and results in lower equilibrium prices.

We now investigate how the existence of network effects influences the stores' market shares. With no loss of generality, assume that store A has the larger market share, (i.e., $a > b$). Differentiating (5) with respect to β yields

$$\frac{\partial \hat{x}^h}{\partial \beta} = \frac{\tau(a-b)}{6(\tau-\beta)^2} > 0. \tag{8}$$

Then, differentiating (5) with respect to α and N yields

$$\frac{d\hat{x}^h}{d\alpha} = \frac{\partial \hat{x}^h}{\partial \beta}\frac{N}{L} = \frac{\tau(a-b)}{6(\tau-\beta)^2}\frac{N}{L} > 0 \quad \text{and} \quad \frac{d\hat{x}^h}{dN} = \frac{\partial \hat{x}^h}{\partial \beta}\frac{\alpha}{L} > 0 \ (< 0) \ \text{ if } \alpha > 0 \ (\alpha < 0). \tag{9}$$

Hence,

Proposition 3

1. *An increase in the preference for the clientele size will increase the market share of the large store;*

2. *An increase in the consumer population per unit of area will*

   (a) *decrease the market share of the large store when preferences exhibit negative network effects;*

   (b) *increase the market share of the large store when preferences exhibit weakly positive network effects.*

Proposition 3 states that when consumers attach a higher valuation to the clientele size, the market share of the store with the initial larger market share tends to increase, whereas the market share of the store with the initial lower market share tends to decrease. Clearly, the opposite occurs when consumers tend to care less about the network size, since stores, selling to a large number of consumers, become less attractive. Also, Proposition 3 says that, when preferences exhibit negative network effects, an increase in population makes the large store less attractive relative to the small store, thereby reducing the market share of the large store. In contrast, when bandwagon effects are (weakly) positive, an increase in the consumer population increases the market share of the already large store, thus reducing the market share of the small one.

Note that Proposition 3 analyzes market shares and not the total number of consumers purchasing from each store. We now analyze how the total number of clients buying from each store is affected when the population increases. From (5) and (8) we have

$$L\frac{dn_A}{dN} = L\frac{d[\hat{x}^h N]}{dN} = \frac{L}{2} + \frac{\tau(a-b)}{6(\tau-\beta)} + \frac{\beta\tau(a-b)}{6(\tau-\beta)^2} > 0 \quad \text{if} \quad 3L(\tau-\beta)^2 + \tau^2(a-b) > 0. \tag{10}$$

which holds since $a > b$. We also need to investigate the sign of dn_B/dN which is given by

$$L\frac{dn_B}{dN} = \frac{d[(L-\hat{x}^h)N]}{dN} = L - \hat{x}^h - N\frac{\partial \hat{x}^h}{\partial \beta}\frac{\alpha}{L}. \tag{11}$$

By (5) and (9) we have that

$$L\frac{dn_B}{dN} = \frac{L}{2} - \frac{\tau(a-b)}{6(\tau-\beta)} - \frac{\beta\tau(a-b)}{6(\tau-\beta)^2} > 0 \ (< 0); \text{ if and only if } 3L(\tau-\beta)^2 > \tau^2(a-b). \tag{12}$$

Notice that for $\alpha < 0$ (hence $\beta < 0$), the condition given in (12) always holds since $3L(\tau - \beta)^2 > 3L\tau^2 > \tau^2 L > \tau^2(a-b)$. The following proposition is readily obtained.

Proposition 4 *An increase in consumer population will*

1. *increase the total number of consumers purchasing from the large store A, and*

2. *increase the total number of consumers purchasing from the small store B if and only if either preferences exhibit negative network effects or* $3L(\tau-\beta)^2 > \tau^2(a-b)$.

Thus, an increase in the population density will always increase the number of consumers buying from the large store whether or not preferences exhibit negative or positive network effects. It will also increase the small store's clientele if preferences exhibit negative network effects since a larger population will make the small store relatively more attractive. Thus, when population density increases it is only possible to have a decrease in the small store's clientele when preferences exhibit positive network effects, since in this case large stores become more attractive relative to small stores.

We also have to study how network effects influence the profits of the two stores. Note that Proposition 2 demonstrated that an increase in α reduces the equilibrium prices of both stores. In addition, Proposition 3 demonstrated that an increase in α decreases the market share of the small store. Hence, an increase in α unambiguously lowers the profit of the small store. In contrast, for the large store (store A) the price declines whereas the market share expands, thereby having two opposite effects working at the same time. However, as shown below, when the two stores are differentiated enough in terms of their location, the profit of the large store increases with α.

Proposition 5 *An increase in consumers' preference for the clientele size will*

1. *reduce the profit of the small store, and*
2. *raise the profit of the large store if and only if*

$$\tau(a-b) > 3L(\tau-\beta).$$

Proof. To prove the second part we need to check under what conditions the profit of store A increases with α. That is, using (6), we need to evaluate

$$\frac{d\pi_A^h}{d\alpha} = \frac{d[p_A^h \hat{x}^h N/L]}{d\alpha}.$$

Note that from (4) and (5) we have

$$\hat{x}^h = \frac{p_A^h}{2(\tau-\beta)}$$

so that

$$\pi_A^h = \frac{(p_A^h)^2 N}{2(\tau-\beta)L}. \tag{13}$$

Now, from (4) we have that $\partial p_A^h/\partial\alpha = -N$, so it can be easily verified that

$$\text{sign}\left[\frac{d\pi_A^h}{d\alpha}\right] = \text{sign}\left[-(\tau-\beta)L + \frac{\tau(a-b)}{3}\right]$$

from which the derived inequality immediately follows. ∎

The impact on the store's profit of an increase in the population size is more complicated because the consumer population is a factor in this function.

Proposition 6

1. *When preferences exhibit negative network effects ($\alpha < 0$), an increase in population will increase the profit of both stores.*

2. *When consumers' preferences exhibit weakly positive network effects ($\alpha > 0$), an increase in population will*

   (a) *reduce the profit of the small store, and*

   (b) *raise the profit of the large store if and only if*

$$\tau(a - b) > 3L(\tau - \beta)\left(\frac{2\beta}{\tau} - 1\right).$$

Proof. The first part follows from previous propositions showing that as the population size increases both stores' prices and clientele size increase. Regarding the second part, as in the proof of Proposition 5, we need to evaluate $d\pi_A^h/dN$ where π_A^h is given in (13) and $\partial p_A^h/\partial N = -\alpha$, which is positive if and only if the above condition holds. ■

Hence, the small store is always hurt by a population increase. However, the large store may increase its profit depending on how much closer to the center it is located relative to its competitor.

Consider the special case of Proposition 6 where the two stores are symmetrically located ($a = b$). Then, both stores always benefit from the increase in population if and only if $\beta < \tau/2$, thus implying that the network effects are 'very' weak.

We conclude the analysis of network effects in the Hotelling model by investigating the welfare of A-customers and B-customers when the total consumer population increases. From (1), the utility of such consumers (located sufficiently far from the marginal consumer $\hat{x}^h$, for a small change not to cause the consumer to switch stores) are proportional to

$$U^A \approx \alpha n_A - p_A^h \quad \text{and} \quad U^B \approx \alpha n_B - p_B^h. \tag{14}$$

Now suppose that the population size increases. When preferences exhibit negative network effect ($\alpha < 0$), Proposition 2 states that both prices increase. Since Proposition 3 shows that the number of A and B-customers increase, all consumers must be worse off. In contrast, when $\alpha > 0$, the welfare of A shoppers increase since p_A falls and n_A rises. Also, Proposition 3 shows that the number of B shoppers may decline and, hence, their welfare may also decrease. More precisely, from (14) we have that

$$\frac{\partial U^B}{\partial N} = \alpha \frac{dn_B}{dN} + \alpha. \tag{15}$$

Accordingly, using Proposition 4 it follows that

Proposition 7 *Assume that the population increases.*

1. *If preferences exhibit negative network effects then all consumers become worse off,*

2. *If consumers' preferences exhibit positive network effects, then*

   (a) *the welfare of consumers loyal to store A increases, and*

   (b) *the welfare of consumers loyal to store B increases if and only if*

$$3L(\tau - \beta)^2 > \frac{\tau^2(a-b)}{2}.$$

Note that there is a third group of consumers, that is, those who switch buying from store B to store A when N rises. In general, it is not possible to determine whether these consumers are better or worse off with an increase in N. However, when $\alpha < 0$, then the welfare of these consumers cannot rise. When $\alpha > 0$, a sufficient condition for these consumers to be better off is that the loyal B-customers are themselves better off.

Regarding location choice by stores, we face the same difficulty as in the standard Hotelling model. When conditions (7) are not satisfied, a mixed strategy price equilibrium exists as in Osborne and Pitchik [1987]. Finding the equilibrium locations in a two-stage game remains an open question when network effects are at work.

Since we would like to investigate the welfare effects of changing the consumer population size, we define social welfare (denoted by W) as the sum of consumers' loss/gains from the network size minus total transportation costs (denoted by T) divided by the total number of consumers. Total transportation costs are given by

$$T \equiv N\tau \left[\int_0^a (a-x)dx + \int_a^{\hat{x}^h} (x-a)dx + \int_{\hat{x}^h}^b (b-x)dx + \int_b^L (x-b)dx \right].$$

Therefore,

$$T = N\tau \left[a^2 + b^2 + \frac{L^2}{2} - bL + \hat{x}^2 - (a+b)\hat{x}. \right] \tag{16}$$

The social welfare function is then given by

$$W = \alpha - \frac{T}{N}. \tag{17}$$

Unlike standard location models, average transportation costs vary here with N because the location of the marginal consumer $\hat{x}$ changes with the population size.

Given (16), it is immediate that our social welfare function increases when $\hat{x}^2 - (a+b)\hat{x}$ decreases with N. When $\alpha > 0$, since the market share of the large store increases with N and since $\hat{x} > L/2$, we easily see that $\hat{x}^2 - (a+b)\hat{x}$ falls with N. Consequently, when $\alpha > 0$, average utility decreases when N increases. In contrast, when $\alpha < 0$, social welfare increases since the market share of the large store declines thereby reducing aggregate transportation cost.

Table 1 below summarizes the effects of an increase in consumer population size on the price (p_i), market share (x_i), number of buyers (n_i), and profit level (π_i) of each store, as well as the welfare of loyal i-store buyers, $i = A, B$.

	Negative ($\alpha < 0$)					**Positive** ($\alpha > 0$)				
Store	p_i	x_i	n_i	π_i	U_i	p_i	x_i	n_i	π_i	U_i
Large Store (A)	$(+)$	$(-)$	$(+)$	$(+)$	$(-)$	$(-)$	$(+)$	$(+)$	$(\pm)$	$(+)$
Small Store (B)	$(+)$	$(+)$	$(+)$	$(+)$	$(-)$	$(-)$	$(-)$	$(\pm)$	$(-)$	$(\pm)$

Table 1: Negative and positive network effects: the effects of a population size increase

2.3 Equilibrium under strong bandwagon effects

We now assume that the network effects are strong, that is, $\beta \geq \tau$. This turns out to have a dramatic impact on the resulting market structure. Indeed, as will be shown, in equilibrium, all the clients patronize a single store. This is in sharp contrast with what we obtained in the foregoing where the two stores split the market.

Assume, indeed, that store A charges a price just low enough to allow it to capture the entire market, i.e., $p_A = \alpha N - \tau(L - a - b)$. Then, it has no incentive to decrease its price since it can no longer increase its market. Furthermore, if it raises its price, even by a trifle, store A loses its whole clientele which now patronizes its rival. Some consumers switch to B, making this store more attractive which in turn leads additional consumers to switch. The outcome of this unraveling process is the bunching of all consumers at store B. Hence,

Proposition 8 *Assume that $\beta \geq \tau$. Then, there are two equilibria:*

1. *in one equilibrium, only store A sells and the equilibrium prices are given by*

$$p_A^* = \alpha N - \tau(L - a - b) \quad \textit{and} \quad p_B^* = 0; \tag{18}$$

2. *in the other equilibrium, only store B sells and the equilibrium prices are given by*

$$p_A^* = 0 \quad \textit{and} \quad p_B^* = \alpha N - \tau(L - a - b). \tag{19}$$

Consequently, a price equilibrium always exists, regardless of the stores' locations. At this equilibrium, *only one store supplies the entire market.* However, the equilibrium price of the active store is lower than the monopoly price because of the threat of entry by the other store, a characterization reminiscent of contestable market theory. Observe that the store that captures the market is not indifferent with respect to its location. On the contrary, Proposition 8 shows that its equilibrium price increases when it moves inward. When the network effects are large compared to the transportation costs, both stores have therefore an incentive to move inward. Actually, store A earns its highest profit when both competitors are located together. The same holds for B. Hence, agglomeration at any point of the market gives each store the highest possible profit if it happens to be chosen by the consumers. In other words, stores want to minimize differentiation when bandwagon effects are strong. In contrast, on average, consumer prefer the stores to be located as far away as possible, that is, at the market endpoints because they face in this case the lowest possible mill price.

From the welfare point of view, the two equilibria are not equivalent. Even though the mill prices are the same, consumers bear lower transportation costs if the equilibrium is that for which the store is closer to the market center. Hence, if the other equilibrium occurs, there is a market failure in that resources are wasted on useless transportation (see Farrell and Saloner [1986] for a related argument). Social welfare is highest when both stores are located at the market center because transportation costs are minimized there no matter which store is patronized by the consumers.

3 The Circular City Model with Free Entry

Consider the Salop [1979] model in which consumers gain utility or disutility when other consumers purchase from the same store. There are M stores located on a circumference of length L, where M is endogenously determined in the model. This is illustrated in Figure 3.

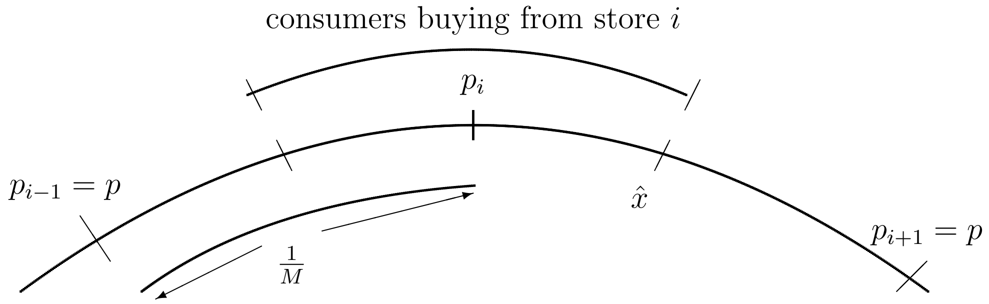

Figure 3: Salop's circular city

We assume equal distance between the stores. Let n_i denote the output level of each store i, $i = 1, \ldots, M$. All stores have identical technologies with linear marginal cost and a fixed cost, given by

$$TC_i(n_i) \equiv cn_i + F; \quad i = 1, \ldots, M. \tag{20}$$

There is a continuum of N consumers uniformly distributed on the circumference. Each consumer buys one unit of the product and bears a transportation cost of τ per unit of distance.

Consider a consumer located at any point x between store i and store $i+1$ in Figure 3. Letting p_i denote the price charged by store i, we define the utility of consumer x by

$$U_x \equiv \begin{cases} \alpha n_i - p_i - \tau x & \text{if he buys from store } i \\ \\ \alpha n_{i+1} - p_{i+1} - \tau(L/M - x) & \text{if he buys from firm } i+1 \end{cases} \tag{21}$$

where α (which could be negative or positive) measures the importance of the clientele size to consumers.

As in Definition 1, we have

Definition 3 *Let $\beta \equiv \alpha N/L$. Then, we say that preferences exhibit* **negative network effects** *if $\alpha < 0$,* **weakly positive network effects** *when $0 < \beta < 2\tau/3$ and,* **strong positive network effects** *when $\beta \geq 2\tau/3$.*

For brevity, we focus only on negative and weakly positive network effects. To simplify our search for a symmetric price equilibrium, we set the prices of all stores except store i equal to $p_j \equiv p$ for all $j \neq i$. We define $\hat{x}_{j,j+1}$ as the distance between store j and the consumer indifferent between buying from j and $j+1$, $j = 1, \ldots, M$. Then, given equal prices $\hat{x}_{j,j+1} = L/(2M)$ for all $j \neq i$, whereas $\hat{x}_{i,i+1}$ satisfies

$$\frac{2\alpha N \hat{x}_{i,i+1}}{L} - p_i - \tau \hat{x}_{i,i+1} = \frac{\alpha N}{L}\left(\frac{L}{M} - \hat{x}_{i,i+1} + \frac{L}{2M}\right) - p - \tau\left(\frac{L}{M} - \hat{x}_{i,i+1}\right).$$

The left hand side shows the welfare of consumer $\hat{x}_{i,i+1}$ when he buys from i, where the total number of buyers is $2N\hat{x}_{i,i+1}$. The right hand side shows the welfare of the same consumer when he buys from $i+1$, where the total number of buyers is $N(L/M - \hat{x}_{i,i+1})$ plus the $NL/(2M)$ consumers located on the other side of consumer $\hat{x}_{i,i+1}$ location. Rearranging yields

$$\hat{x}_{i,i+1}(p_i, p) = \frac{p - p_i}{2\tau - 3\beta} + \frac{L}{2M}. \tag{22}$$

We are looking for a monopolistic competition equilibrium.

Definition 4 *A symmetric monopolistic competition equilibrium is given by the number of stores M° and the prices $p^\circ \equiv p_1^\circ = \ldots = p_M^\circ$, for which:*

Local Oligopoly: *Each store i maximizes its profit at p° when the other stores charge that price:*

$$p^\circ = \arg\max_{p_i} \pi_i(p_i, p^\circ) \equiv 2\hat{x}(p_i, p^\circ)(p_i - c) - F \tag{23}$$

where $\hat{x}(p_i, p^\circ)$ is given in (22).

Free Entry: *The number of stores M° adjusts is such that $\pi_i(p^\circ, p^\circ) = 0$.*

The first and second order conditions are given by

$$0 = \frac{\partial \pi_i(p_i, p^\circ)}{\partial p_i} = \frac{2p - 4p_i}{2\tau - 3\beta} + \frac{L}{M} + \frac{2c}{2\tau - 3\alpha N} \quad \text{and} \quad \frac{\partial^2 \pi_i}{\partial (p_i)^2} = \frac{-4}{2\tau - 3\beta} < 0.$$

Setting $p_i = p^\circ$ yields the symmetric equilibrium price given by

$$p^\circ = c + \frac{(2\tau - 3\beta)L}{2M}. \tag{24}$$

To find the equilibrium number of stores M° we set

$$0 = \pi_i(p^\circ, p^\circ) = \frac{N(p^\circ - c)}{M} - F = \frac{N}{M}\frac{(2\tau - 3\beta)L}{2M} - F$$

implying that the equilibrium number of stores is given by

$$M^\circ = \sqrt{\frac{(2\tau - 3\beta)NL}{2F}}. \tag{25}$$

Substituting into (24) yields

$$p^\circ = c + \frac{1}{2}\sqrt{\frac{2FL(2\tau - 3\beta)}{N}}. \tag{26}$$

Similar to Proposition 2, (26) implies that $dp^\circ/d\alpha < 0$ and $dp^\circ/dN < 0$. Hence,

Proposition 9

1. *An increase in consumers' preference for clientele size α reduces the market price p°.*

2. *An increase in consumer population density always reduces equilibrium prices.*

Comparing Proposition 2 to Proposition 9 reveals that in the linear city model, negative network effects are sufficient for having an increase in population resulting in an increase in prices; however, in the circular city model this result is never obtained. This difference is due to the fixed number of stores in the linear city and free entry in the circular city.

Investigating the effect of a population increase on the equilibrium number of stores, (25) implies that

$$\text{sign}\left[\frac{dM^\circ}{dN}\right] = \text{sign}\;[\tau - 3\beta]. \tag{27}$$

Hence,

Proposition 10 *An increase in population reduces the equilibrium number of stores if and only if $\beta > \tau/3$. Hence, a sufficient condition for an increase in population to increase the number of stores is that preferences exhibit negative network effects ($\beta < 0$).*

Thus, unlike the Salop model where an increase in N always results in additional stores, here an increase in population leads to a reduction in the number of stores when bandwagon effects are high enough relative to the transportation cost. This is because an increase in N exacerbates the bandwagon effect, which in turn makes some of the incumbents more attractive and, therefore, reduces the total number of stores. However, a lower preference for the clientele size (or having negative network effects, say, due to snobbish consumers) brings us back to the standard result where a larger market can support more stores.

We now investigate how the socially optimal number of stores is determined. Our social welfare function is composed of three components: transportation costs, stores' fixed costs, and consumers' gain from clientele sizes.

To calculate the economy's aggregate transportation costs, note that the maximal distance traveled is $L/(2M)$ and the minimal distance traveled is 0. Hence, the average consumer travels a distance of $L/(4M)$. Since there are N consumers, the economy's aggregate transportation bill (as a function of the number of stores) is given by

$$T(M) = \frac{\tau N L}{4M}.$$

Now, in the symmetric equilibrium, each store sells N/M units of the product. Hence, each consumer's network gains (or loss) are $\alpha N/M$. Altogether, the social welfare is given by

$$W(M) \equiv \frac{\alpha N^2}{M} - \frac{\tau N L}{4M} - MF = \frac{(4\beta - \tau)NL}{4M} - MF. \tag{28}$$

Clearly, if $\beta > \tau/4$, the network effect dominates the transportation cost and the socially optimal number of stores is the smallest possible and, therefore, equals one.

However, when $\beta < \tau/4$, differentiating (28) with respect to M yields an optimal number of stores equals to

$$M^* = \sqrt{\frac{(\tau - 4\beta)NL}{4F}}. \tag{29}$$

Comparing (25) with (29) yields $M^* < M^\circ$ if $\beta < 3\tau/2$, which always holds under negative or weakly positive network effects. Accordingly,

Proposition 11 *The equilibrium number of stores always exceeds the socially-optimal number of stores.*

Thus, despite the network effects, just like the original Salop model, the equilibrium number of stores exceeds the socially optimal level.

It is left to investigate what are the effect of an increase in the population on the divergence between the socially optimal number of stores and the equilibrium number. That is, using (25) and (29), we would like to estimate what happens to the ratio given by

$$\frac{M^\circ}{M^*} = \sqrt{\frac{\frac{(2\tau - 3\beta)NL}{2F}}{\frac{(\tau - 4\beta)NL}{4F}}} = \sqrt{2}\sqrt{\frac{2\tau - 3\beta}{\tau - 4\beta}}. \tag{30}$$

Differentiating (30) with respect to N yields

$$\frac{d\left(\frac{M^\circ}{M^*}\right)}{dN} > 0.$$

Hence,

Proposition 12 *Assume that $\beta < \tau/4$. Then, as the population increases, the equilibrium number of stores further diverges from the socially optimal number.*

So far, the population size was assumed to be exogenously given. In what follows, we hypothesize that existing residents of the circular city are engaged in a debate whether to increase the population of the city as in open city models (see Fujita [1989] for a discussion of these models). Therefore, in what follows, we analyze how the welfare of an average citizen of the circular city varies when population slightly increases.

Our average citizen has to travel a distance of $L/(4M^\circ)$. Hence, in view of (21), the utility of an 'average' citizen is the sum of the network gains minus transportation cost minus the price p°, and is given by

$$U(M^\circ(N)) = \frac{\alpha N}{M^\circ} - \frac{\tau L}{4M^\circ} - \frac{(2\tau - 3\beta)L}{2M^\circ} - c = \frac{5\alpha N}{2M^\circ} - \frac{5\tau L}{M^\circ}. \tag{31}$$

Differentiating (31) with respect to N yields

$$\frac{dU(M^\circ(N))}{dN} = \frac{10\alpha M^\circ - 5\alpha N \frac{dM^\circ}{dN}}{4(M^\circ)^2} + \frac{5\tau L}{(M^\circ)^2}\frac{dM^\circ}{dN}.$$

Consequently,

$$\frac{dU(M^\circ(N))}{dN} > 0 \quad \text{if} \quad \frac{dM^\circ}{dN} > \frac{-2\alpha M^\circ}{(4\tau - \beta)L}.$$

Thus,

Proposition 13 *A sufficient condition for having a population increase improving the welfare of existing citizens is that (a) consumer preferences exhibit positive network effects, and (b) the equilibrium number of stores increases with population.*

Table 2 summarizes how an increase in population density affects the circular city.

	$\alpha < 0$			$\alpha > 0$	
p°	M°	U°	p°	M°	U°
$(-)$	$(+)$	$(\pm)$	$(-)$	$(\pm)$	$(\pm)$

Table 2: Effects of population density increase in the circular city

4 Concluding Remarks

We have showed that the introduction of network effects into spatial competition and retailing models has a fairly substantial impact on the market outcome. The most interesting result, maybe, is that small variations in the relative importance of network effects may completely change the market structure from an oligopoly to a single active store under the threat of entry. When our model is reinterpreted within the product differentiation framework, this result sheds new light on the boost in the market share of some products. Furthermore, an increase in the consumer population has different and richer impacts on stores and households than in standard models.

Yet, our approach remains partial in many respects. First, the nonexistence problem in the Hotelling model is not resolved by the presence of network effects. Second, our model is linear in both network effects and transportation costs. It would be instructive to investigate a more general model in order to test the robustness of our main results. Third, the model could be extended in order to include nonobservable attributes when modeled as random variables like in discrete choice theory. Clearly, much work remains to be done in this domain.

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