Knowledge Accumulation and Novelty: Who Should Be Getting the Credit for an Idea?

Chien-fu Chou* and Oz Shy** †

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Abstract

We propose a framework for modelling knowledge accumulation and discoveries. We draw a distinction between novel (original) pieces of knowledge and pieces of knowledge which are deducible from existing knowledge or past discoveries. We show that every deducible piece of knowledge is associated with a unique set of discoveries such that each discovery in this set is indispensable in deducing this piece of knowledge. This framework is useful in analyzing the issues of novelty, patentability, patent infringement, and the value of a patent.

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[0]*State University of New York at Albany, Albany, NY 12222.

[0]**State University of New York at Albany, Albany, NY 12222, and Tel-Aviv University, 69978 Tel-Aviv, Israel.

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"To pursue knowledge is to classify objects."
–Confucius: *The Great Learning*

1. Introduction

We propose a framework which enables us to determine when a piece of knowledge is novel or when it is deducible from other existing pieces of knowledge or previous discoveries. We adopt the standard theory of knowledge and information found in Von Neumann and Morgenstern (1944, Chapter II.8), Savage (1954, Chapters 2 and 6), and Radner (1968) to model knowledge accumulation.[1]

In this note we draw a distinction between novel (original) pieces of knowledge and pieces of knowledge which are deducible from existing knowledge or past discoveries. We show that every deducible piece of knowledge is associated with a unique set of discoveries such that each discovery in this set is indispensable in deducing this piece of knowledge.

The two major issues of a patent system are: (i) determining whether the discoverer of a particular piece of knowledge should be granted with a patent, and (ii) determining whether a certain technology infringes on a specific patent. The framework developed in this paper is useful for analyzing the issues of novelty, patentability, patent infringement, and the value of a patent.

This note is organized as follows. In section 2, we define a novel piece of knowledge and the concepts of a discovery and a history of discoveries. Using the concept of elementary discoveries proposition 1 shows that the novelty of each discovery belonging to a history of discoveries is not destroyed when new discoveries are introduced. Section 3 defines the concept of knowledge deducibility. Proposition 2 shows that for every deducible piece of knowledge there exists a unique minimal set of discoveries from

[1]See also Aumann(1976), Geanakoplos and Polemarchakis (1982), and Milgrom (1981).

which this piece of knowledge can be deduced. Section 4 concludes with the application of the model to the issue of patentability, patent infringement, and licensing fees.

2. A Model of Knowledge Accumulation and Discoveries

We denote by Ω the space of the states of the world. A (known or unknown) piece of knowledge is a set $A \subset \Omega$. A know-how is represented by a finite σ-field of subsets of the state space Ω. Since for every finite field there corresponds a unique finite partition which generates the field and vice versa, we refer to both as representing a know-how.

Given k pieces of knowledge $A_1, \ldots, A_k$, there exists a smallest field of subsets of Ω denoted by $\mathcal{F}(A_1, \ldots, A_k)$ such that $A_1, \ldots, A_k \in \mathcal{F}(A_1, \ldots, A_k)$. That is, all the A_j's are measurable with respect to $\mathcal{F}(A_1, \ldots, A_k)$. We call $\mathcal{F}(A_1, \ldots, A_k)$ the know-how generated by the k pieces of knowledge $A_1, \ldots, A_k$.

DEFINITION 1 A piece of knowledge A is said to be *novel* with respect to economy's know-how $\mathcal{F}(A_1, \ldots, A_k)$ if $A \notin \mathcal{F}(A_1, \ldots, A_k)$. That is, if A is not measurable with respect to $\mathcal{F}(A_1, \ldots, A_k)$.

Intuitively, only novel pieces of knowledge "improve" the economy's know-how. Formally, only a novel piece of knowledge refines the existing field representing the economy's know-how. Thus, we now make the following definition.

DEFINITION 2 A *history of discoveries* is a sequence of pieces of knowledge $A_1, \ldots, A_T$ such that A_t is novel with respect to $\mathcal{F}(A_1, \ldots, A_{t-1})$, $t = 1, \ldots, T$. The piece of knowledge A_t in the sequence is called the t-th discovery.

Thus, a discovery is a piece of knowledge which is novel with respect to previous discoveries in the history. A history of discoveries $A_1, \ldots, A_T$, defines a sequence of know-how $\{\mathcal{F}_t\}_{t=0}^T$ where $\mathcal{F}_t \equiv \mathcal{F}(A_1, \ldots, A_t)$ and $\mathcal{F}_0 \equiv \{\emptyset, \Omega\}$. The σ-field $\mathcal{F}_t$ represents the economy's know-how after the t-th discovery. We now denote by $\Pi_t \equiv$

$\{P_1^t, \ldots, P_{n_t}^t\}$ the unique partition induced by $\mathcal{F}_t$. Loosely speaking, each set $P_i \in \Pi_t$ is a building block for the economy's know-how after the t-th discovery occurs. Each piece of knowledge in $\mathcal{F}_t$ is a union of such building blocks.

DEFINITION 3 A_{t+1} is called an *elementary discovery* if A_{t+1} is a proper subset of P_i^t for some i, $1 \leq i \leq n_t$.

Thus, an elementary discovery refines one and only one set (a building block) in the partition Π_t. In general, a non-elementary discovery may contain some building blocks and may refine more than one building block. Figure 1 illustrates a non-elementary discovery A_{t+1} which contains P_1^t and refines the sets P_2^t and P_3^t. Thus, the discovery A_{t+1} introduces two (unrelated) new (novel) pieces of knowledge, which can be viewed as the "contribution" of this discovery to the economy's know-how. Therefore, the discovery A_{t+1} can be decomposed into two elementary discoveries (the sets B and C in figure 1). Thus, there is no loss of generality by restricting our analysis to histories consisting of only elementary discoveries.

The following proposition shows that if a new discovery occurs, early discoveries remain novel in the sense that taking away old discoveries will result in a loss of knowledge (the σ-field becomes coarser). The proof is found in the appendix.[2]

Proposition 1 *Every discovery belonging to a history of elementary discoveries is novel with respect to all other discoveries belonging to the economy's history. Formally,* $A_t \notin \mathcal{F}(A_1, \ldots, A_{t-1}, A_{t+1}, \ldots, A_T)$ *for every* $t = 1, 2, \ldots, T$.

Proposition 1 implies that if we change the order of the elementary discoveries of a given history, the novelty of each discovery is not destroyed.[3]

[2]This proposition may not hold for histories consisting of non-elementary discoveries.

[3]Note that a reordering will make some discoveries not elementary with respect to the new ordering.

3. Deduction of Knowledge

Consider an economy with a given history of elementary discoveries given by $(A_1, \ldots, A_T)$. We need the following definitions. Let J be a set of indices, $J = \{j_1, \ldots, j_r\} \subset \{1, 2, \ldots, T\}$. Define $\mathcal{F}_J \equiv \mathcal{F}(A_{j_1}, \ldots, A_{j_r})$. Thus, each set of indices J corresponds to a subset of discoveries and the σ-field $\mathcal{F}_J$ they generate. The following definition is based on the assumption that agents have unbounded rationality in the sense that they can perform all the necessary set operations in order to deduce a piece of knowledge.[4]

DEFINITION 4

1. A piece of knowledge A is said to be *deducible* from a set of discoveries J if $A \in \mathcal{F}_J$.
2. A piece of knowledge A is said to be *known* if it is deducible from $\{1, \ldots, T\}$.
3. A is said to *depend* on a set of discoveries J if A is deducible from J but not from any proper subset of J. That is, $A \in \mathcal{F}_J$ but $A \notin \mathcal{F}_{J'}$ for every $J' \subset J$ and $J' \neq J$.

It follows from definition 4 that a discovery is a piece of knowledge which is not deducible from previous discoveries. Clearly, if A is deducible from J, then A is not novel with respect to $\mathcal{F}_J$. Also, a known piece of knowledge is one which is deducible from the entire history.

There is the question of whether a piece of (known) knowledge can depend on different sets of discoveries. That is, if the idea behind a piece of knowledge can be credited to different sets of discoveries in the history. The following proposition shows that this is not the case.

[4] This kind of deduction is rather extreme in the sense that all mathematical theorems and algorithms are deducible and therefore, as discussed in the next section, are not patentable. Assuming bounded rationality is rather complex and will not be discussed here.

Proposition 2 *If A is known, then there exists a unique set of indices $J \subset \{1, \ldots, T\}$ such that A depends on J.*

The following lemma says that if a piece of knowledge can be deduced from two different sets of discoveries then it can be deduced from the intersection of the two sets. The lemma is proved in the appendix.

Lemma. *For any sets of indices J and J', $\mathcal{F}_J \cap \mathcal{F}_{J'} = \mathcal{F}_{J \cap J'}$. That is, a piece of knowledge is deducible from J and from J' if and only if it is deducible from $J \cap J'$.*

Proof of Proposition 2:
Define $\mathcal{J}_A \equiv \{J' \subset \{1, 2, \ldots, T\} : A \in \mathcal{F}_{J'}\}$. Clearly, $A \in \bigcap_{J' \in \mathcal{J}_A} \mathcal{F}_{J'}$. Define $J \equiv \bigcap_{J' \in \mathcal{J}_A} J'$. By the lemma we have that $\mathcal{F}_J = \bigcap_{J' \in \mathcal{J}_A} \mathcal{F}_{J'}$. Therefore, the unique set of indices J is obtained. *Q.E.D.*

Proposition 2 shows that each piece of known knowledge depends on a unique set of discoveries. Therefore, we denote by J_A the unique set of discoveries on which a piece of known knowledge A depends. Also, if a discovery j belongs to J_A, then A is not deducible without the discovery j.

Recalling that know-how is a σ-field, the following definition is used to identify when know-how is "available."

DEFINITION 5 Know-how $\mathcal{F}$ is said to be *comprehendible* if every piece of knowledge $A \in \mathcal{F}$ is known. That is, $\mathcal{F} \subset \mathcal{F}_T$.

Thus, every piece of knowledge in comprehendible know-how $\mathcal{F}$ can be deduced from the existing discoveries. Proposition 2 implies that there exists a smallest collection of discoveries denoted by $J_{\mathcal{F}}$ such that every piece of knowledge in $\mathcal{F}$ can be deduced from $J_{\mathcal{F}}$. We can say that $J_{\mathcal{F}}$ is the set of discoveries which are *indispensable* to the know-how $\mathcal{F}$. It is clear that $J_{\mathcal{F}} = \bigcup_{A \in \mathcal{F}} J_A$. Notice that $\mathcal{F} \subset \mathcal{F}_{J_{\mathcal{F}}}$ and in general $\mathcal{F} \neq \mathcal{F}_{J_{\mathcal{F}}}$.

4. Conclusion: An Application to Patents, Patent Infringement, and Licensing Fees

The previous sections develop a framework to model knowledge and discoveries and offer a method of distinguishing novel (original) knowledge from deducible (non-original) knowledge. In addition, this framework allows us to trace back the precise sources of each deducible piece of knowledge. We now show how this framework can be used to define the concept of patentability and patent infringement.

Patent laws forbid the use of know-how which relies on patents without the agreement of the patent holders. This raises the issues of how to determine when a piece of knowledge is novel (original) and if not, what are exactly the pieces of original knowledge which are used in deducing this piece. The first issue can be easily analyzed in the present framework. Given the economy's history of discoveries $(A_1, \ldots, A_T)$ and the corresponding know-how $\mathcal{F}_T$, a piece of knowledge A is said to be *patentable* if it is novel with respect to $\mathcal{F}_T$. Thus, a piece of knowledge is patentable if it cannot be deduced from other discoveries.[5] If patents are awarded for a given finite length of time and if the discoveries $(A_1, \ldots, A_T)$ are ordered according to the time of discoveries, then there exists a $\hat{t}$ such that the discoveries $(A_{\hat{t}}, \ldots, A_T)$ are patented.

The second issue can be analyzed by associating each production technology with a specific information structure which is captured by the producer's know-how, see Radner (1968).[6] We say that a technology is *feasible* if it is associated with comprehendible know-how. Proposition 2 implies that comprehendible know-how depends on a unique set of discoveries which are indispensable to this know-how. If this set

[5] One way to interpret a discovery A is to think of it as a device which can tell whether the event A occurs.

[6] However, here information sets are changing. Note that there can be several technologies associated with the same know-how.

of discoveries contains some patented discoveries, then a technology associated with this know-how can be used only under a licensing agreement from the patent holders. Otherwise, we say that this technology infringes on patented discoveries.[7]

A natural question to ask at this point is as follows. Suppose that a technology infringes on one patented discovery. Then, how can we calculate the contribution of the patented discovery to this technology. More precisely, what should be the (licensing) fee paid by the technology user to the patent holder? To determine the value of each discovery we need to make Ω a probability space, which we denote by $(\Omega,\ \mathcal{B},\ \mathcal{P})$, where $\mathcal{B}$ is σ-field and $\mathcal{P}$ is a probability measure. We restrict all the knowledge pieces to be $\mathcal{B}$-measurable. Consider know-how $\mathcal{F}$ and let $J_{\mathcal{F}} \equiv \{j_1, j_2, \ldots, j_m\}$ with $j_{i-1} < j_i$. Define $\mathcal{F}(k) \equiv \mathcal{F} \cap \mathcal{F}(j_1, \ldots, j_k)$ and $\mathcal{F}(0) \equiv \{\Omega, \emptyset\}$. Thus, $\mathcal{F}(0) \subset \mathcal{F}(1) \subset \cdots \subset \mathcal{F}(m) = \mathcal{F}$ is an increasing sequence of σ-fields. Suppose that the *unit* production cost is a function of input combination and the state $\omega \in \Omega$, which is denoted by $c(x(\omega),\ \omega)$, where $x(\omega)$ is interpreted as the input employment level at state ω, and x is a measurable function with respect to the available know-how $\mathcal{F}(k)$. The producer chooses an input function $x(\omega)$ to

$$\min_{x(\omega) \text{ is } \mathcal{F}(k)\text{-measurable}} E\left[c(x(\omega);\ \omega) \mid \mathcal{F}(k)\right] \equiv c(\mathcal{F}(k)).$$

It is clear that $c(\mathcal{F}(0)) \geq c(\mathcal{F}(1)) \geq \cdots \geq c(\mathcal{F}(m))$. Now, observe that the cost reduction attributed to the discovery j_k can be calculated as the difference $c(\mathcal{F}(k)) - c(\mathcal{F}(k-1))$, which can be the per-unit (licensing) fee paid to the j_k's patent holder. Note that here the order of discoveries does affect the fee collected by each patent holder.

Appendix

[7]See Green and Scotchmer (1989) which looks at patentability from a product improvement point of view.

Proof of Proposition 1.

With no loss of generality we set $t = 1$ and show that A_1 is novel with respect to $\mathcal{F}(A_2, \dots, A_T)$. The proposition is proved by induction. Clearly, $A_1 \notin \mathcal{F}_0$. Suppose that $A_1 \notin \mathcal{F}(A_2, \dots, A_k)$. We want to show that $A_1 \notin \mathcal{F}(A_2, \dots, A_{k+1})$. Let $\Pi' = (P'_1, \dots, P'_k)$ be the partition corresponding to field $\mathcal{F}(A_2, \dots, A_k)$. By the induction hypothesis, $A_1 \notin \mathcal{F}(A_2, \dots, A_k)$, implying that A_1 can be expressed uniquely as $A_1 = B \cup D$, where $B \in \mathcal{F}(A_2, \dots, A_k)$, D is a proper subset of P'_i for some $i \in \{1, \dots, k\}$, and $B \cap P'_i = \emptyset$. Consider the (elementary) discovery A_{k+1}. There are only three possibilities: $A_{k+1} \subset D$, $A_{k+1} \subset P'_i \setminus D$, and $A_{k+1} \subset P'_j$ for $j \neq i$. In all of the three cases, A_1 is not measurable with respect to $\mathcal{F}(A_2, \dots, A_{k+1})$. *Q.E.D.*

Proof of the Lemma:

Let $J = (J \cap J') \cup \{j_1, \dots, j_k\}$ and $J' \cap \{j_1, \dots, j_k\} = \emptyset$. The proof proceeds by induction. To simplify the notation, we define $J_h = (J \cap J') \cup \{j_1, \dots, j_h\}$ and $J_0 = J \cap J'$. First observe that $\mathcal{F}_{J_0} \cap \mathcal{F}_{J'} = \mathcal{F}_{J \cap J'}$. Suppose that $\mathcal{F}_{J_{h-1}} \cap \mathcal{F}_{J'} = \mathcal{F}_{J \cap J'}$. We want to show that $\mathcal{F}_{J_h} \cap \mathcal{F}_{J'} = \mathcal{F}_{J \cap J'}$.

(i) Since all the discoveries are elementary, $A_{j_h} = E \cup G$ where $E \in \mathcal{F}_{J_{h-1}}$, G is a proper subset of a set P'' in the partition corresponding to the finite field $\mathcal{F}_{J_{h-1}}$, and $E \cap P'' = \emptyset$. By proposition 1, $A_{j_h} \notin \mathcal{F}_{J_{h-1} \cup J'}$ implying that $G,\ P'' \setminus G \notin \mathcal{F}_{J_{h-1} \cup J'}$.

(ii) Every $C \in \mathcal{F}_{J_h} \setminus \mathcal{F}_{J_{h-1}}$ can be represented as $C = B \cup D$ where $B \in \mathcal{F}_{J_{h-1}}$, $D = G$ or $D = P'' \setminus G$, and $B \cap D = \emptyset$. By *(i)*, $C \notin \mathcal{F}_{J_{h-1} \cup J'}$ and therefore, $C \notin \mathcal{F}_{J'}$. Thus, $\mathcal{F}_{J_h} \cap \mathcal{F}_{J'} = \mathcal{F}_{J \cap J'}$. *Q.E.D.*

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