

Last Name (Please PRINT): .....

First Name (PRINT): .....

Your I.D. Number: .....

INSTRUCTIONS (please read!)

1. Please make sure that you have 9 pages, including this page. Complaints about missing pages will not be accepted.
2. Please answer all the questions. You are not allowed to use any course material. Calculators are permitted.
3. Maximum Time Allowed: 3 hours.
4. Your grade depends on the arguments you develop for supporting your answers. Each answer must be justified by using a logical argument consisting of a model/graph. An answer with no justification will not be given any credit.
5. You must provide all the derivations leading you to a numerical solution. Please do *not* use any “formulas” developed in class. You need to drive them by yourself.
6. When you draw a graph, make sure that you label the axes with the appropriate notation.
7. Maximum Score: 100 Points
8. Budget your time. If you cannot answer a certain question, skip it and go to the next one.
9. Please always bear in mind that “somebody” has to read and understand your handwriting. Please make sure that your ink is ‘visible’ and that your sentences are properly organized and fit into the designated blank space. If you think that your handwriting is poor, please print each word!
10. Good Luck !

(1) Consider a duopoly (two-firm) computer industry producing two brands named *Artichoke* (Brand *A*), and *Banana* (Brand *B*). Assume that each computer costs \$2 to produce. Let  $p_A$  denote the price charged by Artichoke, and  $p_B$  the price charged by Banana.

There are 100 consumers who are brand *A*-oriented consumers, and 100 consumers who are brand *B*-oriented consumers. Let  $q_A$  be the number of consumers who purchase brand *A*, and  $q_B$  the number of consumers who purchase brand *B*. Formally, the utility of *A*-oriented and *B*-oriented consumers are given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} 0.5q_A - p_A & \text{buys } A ; A \text{ is incompatible} \\ 0.5q_B - p_B - 300 & \text{buys } B ; B \text{ is incompatible} \\ 0.5(q_A + q_B) - p_A & \text{buys } A ; A \text{ is } B\text{-compatible} \\ 0.5(q_A + q_B) - p_B - 300 & \text{buys } B ; B \text{ is } A\text{-compatible,} \end{cases} \quad (1)$$

and

$$U_B \stackrel{\text{def}}{=} \begin{cases} 0.5q_A - p_A - 300 & \text{buys } A ; A \text{ is incompatible} \\ 0.5q_B - p_B & \text{buys } B ; B \text{ is incompatible} \\ 0.5(q_A + q_B) - p_A - 300 & \text{buys } A ; A \text{ is } B\text{-compatible} \\ 0.5(q_A + q_B) - p_B & \text{buys } B ; B \text{ is } A\text{-compatible.} \end{cases}$$

(1a) [8 pts.] Calculate the undercut-proof equilibrium prices assuming that the computer brands are incompatible. *Hint*: First make sure that you know to define price-undercutting considering the fact that each unit costs \$2 to produce.

(1b) [2 pts.] Calculate the equilibrium profit level of each firm when the brands are incompatible.

(1c) [8 pts.] Calculate the undercut-proof equilibrium prices assuming that the computer brands are compatible.

(1d) [2 pts.] Calculate the equilibrium profit level of each firm when the brands are compatible.

**(2)** Consider a world with two countries labeled  $N$  (for North) and  $S$  (for South). Country  $N$  has  $\eta_N$  consumers who wish to place at most one international phone call to country  $S$ . Country  $S$  has  $\eta_S$  consumers who wish to place at most one phone call to country  $N$ . Assume that  $\eta_N > \eta_S$ .

Let  $p_k$  denote the price of a phone call from country  $k$  as charged by the country  $k$ 's carrier,  $k = N, S$ . Each potential consumer has a valuation of  $\beta > 0$  for placing this phone call, meaning that the utility function of a consumer in country  $k$  is given by

$$U_k \stackrel{\text{def}}{=} \begin{cases} \beta - p_k & \text{if makes an international call} \\ 0 & \text{if does not make an international call.} \end{cases}$$

Let  $a$  denote the international access charge (settlement rate), which is the payment each carrier makes to the foreign carrier for carrying the phone call to its final destination in the foreign country. Then the profit of each national phone company is composed of profit generated from sales of international phone calls and the collection of access fees from incoming international phone calls. Thus,

$$\pi_N \stackrel{\text{def}}{=} (p_N - a)\eta_N + a\eta_S, \quad \text{and} \quad \pi_S \stackrel{\text{def}}{=} (p_S - a)\eta_S + a\eta_N.$$

Suppose now that the phone industry in country  $N$  is fully competitive, hence the price of an international phone call from  $N$  to  $S$  is  $p_N = a$ , where  $a$  is the negotiated access charge. Also, suppose that the phone industry in country  $S$  is a monopoly, hence the price of a phone call from country  $S$  to  $N$  is  $p_S = \beta$ . Answer the following questions.

**(2a) [5 pts.]** Let  $a_N$  be the access charge that maximizes  $\pi_N$ , and let  $a_S$  be the access charge that maximizes  $\pi_S$ . Calculate  $a_N$  and  $a_S$ . Show your calculations.

**(2b) [5 pts.]** Using the bargaining rule  $\hat{a} = (a_N + a_S)/2$ , calculate the net flow of money transferred from company  $N$  to company  $S$ . That is, calculate  $T_{N \rightarrow S}$ . Show your calculations!

**(2c) [10 pts.]** Answer questions (2a) and (2b) assuming that the phone industry in country  $N$  and in country  $S$  are both competitive. Show your calculations.

**(3)** Suppose that there are 4 possible TV program types indexed by  $i = 1, 2, 3, 4$ . For example, type 1 could be a talk show, type 2 could be the news, type 3 could be a fashion show, and type 4 could be a sports program. Each type of program  $i$  is watched by  $\eta_i$  viewers. Suppose that  $\eta_1 = 400$ ,  $\eta_2 = 150$ ,  $\eta_3 = 100$ , and  $\eta_4 = 80$ .

Assume that (i) Programs are to be aired in prime time only; hence each broadcasting station can air at most one program type. (ii) If several stations choose to air the same program type, then the program's viewers are evenly split among the stations.

There are three broadcasting stations indexed by  $j = A, B, C$ . Production is costless. Each station earns a profit of \$1 on each viewer, so each station attempts to maximize the number of viewers. We denote by  $p_j \in 1, 2, 3, 4$  the action (program type) chosen by station  $j$ .

**(3a) [10 pts.]** Find which type of program will be broadcasted by each station in a Nash equilibrium. You must PROVE your answer using the definition of a Nash equilibrium.

**(3b) [10 pts.]** Suppose that each viewer gains a utility of  $U_i = \beta$  if the program of his choice is aired, and  $U_i = 0$  if the program of his choice is not aired. Define the social welfare function  $W$  by the sum of viewers' utilities. Find the allocation of programs among the three networks that would maximize social welfare. Prove your answer!

(4) Consider a monopoly cable-TV operator providing a service to three consumers by transmitting three channels: CNN, BBC, and SHOP(ping). Assume that the monopoly's production (transmission) is costless. The Table below shows the valuation (maximum willingness to pay) of each consumer for each channel.

Consumer	CNN	BBC	SHOP
1	5	1	2
2	5	1	5
3	1	5	2

**Table 1:** Consumers' valuation of three channels.

(4a) [10 pts.] Calculate the profit-maximizing prices assuming that the monopoly must sell each channel separately.

(4b) [5 pts.] Calculate the profit-maximizing price assuming that the monopoly sells all the channels in a single package.

(4c) [5 pts.] Suppose now that the monopoly can package channels any way it wants to. Which combination of packages maximize the monopoly's profit?

(5) Consider a technology-adoption game played by two users (or firms) displayed in following table.

		User <i>B</i>	
		NEW TECHNOLOGY	OLD TECHNOLOGY
User <i>A</i>	NEW	3	1
	OLD	0	2

(5a) [10 pts.] Which technology will be adopted by each user in Nash equilibrium. That is, find the Nash equilibrium(ia) for this game (if they exist). Prove your answer!

(5b) [10 pts.] Does the outcome (New, New) constitute a case of *excess momentum*? Explain your answer using the definition of the term.



***Scratch Paper***

This page will NOT be read by the instructor !

***THE END***