file=net-w09-final.tex

Last Name (Please PRINT): .....

First Name (PRINT): .....

Your UM I.D. Number: .....

INSTRUCTIONS (please read!)

1. Please make sure that you have 8 pages, including this page. Complaints about missing pages will not be accepted.
2. Please answer all the questions. You are not allowed to use any course material. Calculators are permitted.
3. Maximum Time Allowed: 2 hours (8:00–10:00).
4. Your grade depends on the arguments you develop for supporting your answers. Each answer must be justified by using a logical argument consisting of a model/graph. An answer with no justification will not be given any credit.
5. You must provide all the derivations leading you to a numerical solution.
6. When you draw a graph, make sure that you label the axes with the appropriate notation.
7. Maximum Score: 100 Points
8. Budget your time. If you cannot answer a certain question, skip to the next one.
9. Please always bear in mind that "somebody" has to read and understand your handwriting. Please make sure that your ink is "visible" and that your sentences are properly organized and fit into the designated blank space. If you think that your handwriting is poor, please print each word!
10. Good Luck !

Instructor's use only

Problem #	1	2	3	4	5	6	Total
Maximum	20	15	15	15	15	20	100
Points							

(1) In the neighborhood where you live, there are $\eta = 3$ residents who are connected to a single Internet service provider (ISP). The ISP does not bear any cost of production. The capacity of this network is $\bar{Q} = 30$ Mbps (maga-byte per second). Let p denote the price the ISP charges each resident for each 1 Mbps of usage (say, for the amount of downloading). Let q_i denote the demand of resident i (also measured in Mbps). Each resident i ($i = 1, 2, 3$) has a utility function given by

$$U_i = \sqrt{q_i} - \frac{Q}{\bar{Q}} - pq_i = \sqrt{q_i} - \frac{q_1 + q_2 + q_3}{30} - pq_i,$$

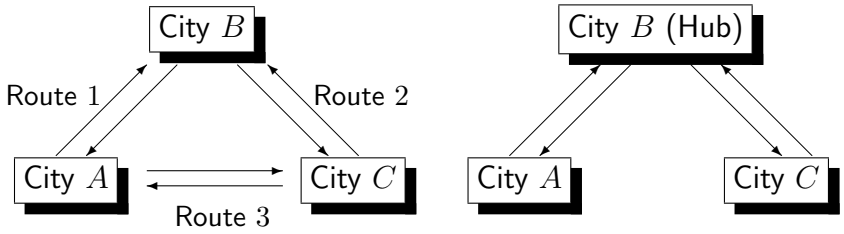
where $Q/\bar{Q} = Q/30$ is the disutility generated by congestion.

(1a) [7 points] Find how much Internet is demanded by each resident i in a Nash equilibrium assuming that the Internet is provided for free ($p = 0$). Formally, compute q_i. Does not network operate below or above its capacity?

(1b) [7 points] Compute the socially-optimal Internet usage level of each resident. Does not network operate below or above its capacity when each resident uses the Internet at the socially-optimal level?

(1c) [6 points] Compute the price that the ISP should charge (per Mbps) in order to induce all residents to demand Internet at the socially-optimal level.

(2) A single airline company serves 3 cities as illustrated in the following figure.



The number of passengers on each route are $\eta_1 = 1800$, $\eta_2 = 700$, and $\eta_3 = 1800$. The costs of operating a flight on each route are is given by the functions $c(q_1) = 50 + \sqrt{q_1}$, $c(q_2) = 40 + \sqrt{q_2}$, and $c(q_3) = 50 + \sqrt{q_3}$, where q_i is the actual number of passengers flying on route i.

The airline considers two alternative networks of operations (displayed in the above figure): A fully-connected (FC), and a Hub-and-Spoke (with a hub in city B).

(2a) [5 points] Calculate which network of operation (FC or HS) minimizes the airline's cost. Show your calculation!

(2b) [10 points] Suppose that the airline has already decided to operate a Hub-and-Spoke network. Which city should be chosen to be the hub? Prove your answer by computing (separately) the total cost of operation when city A, B, or C are chosen to be the hub.

(3) Consider the discrete-demand model of the market for telecommunication, with three types of consumers, indexed by i, $i = 1, 2, 3$. There are 300 consumers of each type (hence, total 900 potential subscribers). Let q denote the number of people connecting to this telecommunication service, and let p denote the connection fee. The utility of each type i consumer is given by

$$U_1 \stackrel{\text{def}}{=} \begin{cases} 2q - p & \text{if connected} \\ 0 & \text{otherwise,} \end{cases} \qquad U_2 \stackrel{\text{def}}{=} \begin{cases} 3q - p & \text{if connected} \\ 0 & \text{otherwise,} \end{cases} \qquad U_3 \stackrel{\text{def}}{=} \begin{cases} 4q - p & \text{if connected} \\ 0 & \text{otherwise,} \end{cases}$$

(3a) [5 points] Draw the aggregate demand for telecommunication services. Explain your result!

(3b) [5 points] Suppose that the connection fee is $p = 600$. Find the critical mass associated with this level of connection fee.

(3c) [5 points] Suppose that this market is serviced by a single monopoly firm who has no production costs for connecting people to this service. Find the monopoly's profit-maximizing connection fee and its profit level.

(4) Consider a scheduling competition between two broadcasting networks labeled A and B which broadcast only morning news. There are 1000 potential viewers. All viewers work full time and therefore can watch the morning news only before they leave for work (and not after). More precisely,
300 viewers prefer to watch the news at 06:00 (and not after 06:00 because they must leave for work).
500 viewers prefer to watch the news at 07:00 (and not after 07:00 because they must leave for work).
100 viewers prefer to watch the news at 08:00 (and not after 08:00 because they must leave for work).
100 viewers prefer to watch the news at 09:00 (and not after 09:00 because they must leave for work).

Each network must pick the time for its morning news. The profit of network A is $\pi_A(t_A, t_B) = \rho n_A$ where ρ is the revenue per viewer collected from the advertisers, and n_A is the equilibrium number of viewers who watch A. Similarly, $\pi_B(t_A, t_B) = \rho n_B$.

(4a) [5 points] Let t_A denote the broadcasting time of network A, and t_B of network B. Compute the networks' best response functions.

(4b) [5 points] Conclude which broadcasting times $\langle t_A, t_B \rangle$ constitute a Nash equilibrium and compute the equilibrium profit levels $\pi_A(t_A, t_B)$ and $\pi_B(t_A, t_B)$.

(4c) [5 points] Suppose now that broadcaster B went out of business (bankrupt). Which broadcasting time will be chosen by network A? Prove your answer.

(5) Consider the demand for a telecommunication service subscription in which consumer types are indexed by x on the interval $[0,1]$. Suppose there are $\eta = 240$ consumers of each consumer type x. Assume that the utility function of each consumer x, $x \in [0,1]$ is given by

$$U_x = \begin{cases} (2-3x)q^e - p & \text{if she subscribes} \\ 0 & \text{if she does not subscribe,} \end{cases}$$

where p is the subscription price, and q^e is the expected number of subscribers.

(5a) [5 points] Formulate and draw the aggregate inverse demand function for this service. Characterize the subscription level under which consumers' willingness to pay reaches the highest level.

(5b) [5 points] Compute the critical mass, q^{cm}, at the subscription price $p_0 = \$60$.

(5c) [5 points] Compute the profit-maximizing number of connections and the corresponding price charged by a monopoly service provider. Assume that this provider does not bear any production cost.

(6) In a small island in South-East Asia there are 60 native Bengali speakers and 40 native Hindi speakers. The language school on this island teaches 3 separate classes: Bengali (language B), Hindi (language H), and English (language E). The cost of learning English is $p_E = \$20$. The cost of learning Bengali is $p_B = \$50$, and the cost of learning Hindi is $p_H = \$50$.

Denote by n_{BH} the number of Bengali speakers who learn Hindi; by n_{BE} the number of Bengali speakers who learn English; n_{HB} the number of Hindi speakers who learn Bengali; n_{HE} the number of Hindi speakers who learn English. The utility functions of a Bengali and an Hindi native speakers are

$$U_B = \begin{cases} 60 + n_{HB} & \text{does not study} \\ 100 - 50 & \text{studies Hindi} \\ 60 + n_{HB} + n_{HE} - 20 & \text{studies English} \end{cases} \quad \text{and} \quad U_H = \begin{cases} 40 + n_{BH} & \text{does not study} \\ 100 - 50 & \text{studies Bengali} \\ 40 + n_{BH} + n_{BE} - 20 & \text{studies English} \end{cases}$$

(6a) [5 points] State whether the following statement is True or False: There exists an equilibrium in which all the island's residents learn English. You must prove your answer.

(6b) [5 points] State whether the following statement is True or False: There exists an equilibrium in which all native Hindi speakers learn Bengali whereas Bengali native speakers don't learn any new language. You must prove your answer.

(6c) [10 points] Compute aggregate consumer welfare levels corresponding to the following four separate situations:

I. All residents learn English.
II. All Bengali native speakers learn Hindi.
III. All Hindi native speakers learn Bengali.
IV. No one learns any new language.

Conclude which situation yields the highest aggregate consumer welfare.

THE END