Solution to Set # 1: The Network Externalities Approach

(a) (i) Firm B maximizes p_B subject to the constraint

$$\pi_A = (p_A - 2) \times 100 \geq (p_B - 2 - 300 + 0.5 \times 100) \times 200$$

Firm A maximizes p_A subject to the constraint

$$\pi_B = (p_B - 2) \times 100 \geq (p_A - 2 - 300 + 0.5 \times 100) \times 200$$

Solving 2 equations with two variables yields $\boxed{p_A^I = p_B^I = 502}$.

(ii)

$$\pi_A^I = (p_A^I - 2) \times 100 = 50,000 \quad \text{and} \quad \pi_B^I = (p_B^I - 2) \times 100 = 50,000$$

(iii) When the machines are compatible, undercutting does not affect the network component in consumers' utility (since each consumer can interact with 200 consumers regardless of which brand he buys). Hence, Firm B maximizes p_B subject to the constraint

$$\pi_A = (p_A - 2) \times 100 \geq (p_B - 2 - 300) \times 200$$

Firm A maximizes p_A subject to the constraint

$$\pi_B = (p_B - 2) \times 100 \geq (p_A - 2 - 300) \times 200$$

Solving 2 equations with two variables yields $\boxed{p_A^I = p_B^I = 602}$.

(iv)

$$\pi_A^I = (p_A^I - 2) \times 100 = 60,000 \quad \text{and} \quad \pi_B^I = (p_B^I - 2) \times 100 = 60,000$$

(b) (i) Service provider A maximizes p_A subject to

$$\pi_B = 120 p_B \geq 240(p_A - 60 + 0.5 \cdot 120).$$

Similarly, service provider B maximizes p_B subject to

$$\pi_A = 120 p_A \geq 240(p_B - 90 + 0.5 \cdot 120).$$

Solving the two equation under equality yields

$$p_A^I = 20, \quad p_B^I = 40, \quad \pi_A^I = 120 \cdot 20 = 2400, \quad \text{and} \quad \pi_B^I = 120 \cdot 40 = 4800,$$

where superscript I indicates equilibrium values under incompatible networks.

(ii) Service provider A maximizes p_A subject to

$$\pi_B = 120p_B \geq 240(p_A - 60).$$

Similarly, service provider B maximizes p_B subject to

$$\pi_A = 120p_A \geq 240(p_B - 90).$$

Solving the two equation under equality yields

$$p_A^C = 140, \quad p_B^C = 160, \quad \pi_A^C = 120 \cdot 140 = 16,800, \quad \text{and} \quad \pi_B^C = 120 \cdot 160 = 19,200,$$

where superscript C indicates equilibrium values under incompatible networks.

(iii) Under incompatible networks:

$$U_A^I = \frac{1}{2}120 - 20 = 40 \quad \text{and} \quad U_B^I = \frac{1}{2}120 - 40 = 20.$$

Under compatible networks:

$$U_A^C = \frac{1}{2}240 - 140 = -20 < 40 \quad \text{and} \quad U_B^C = \frac{1}{2}240 - 160 = -40 < 20.$$

Hence, all types of consumers are worse off under compatible networks. This is because of the very high prices both service providers charge when they sell compatible services compared with the prices they charge when they sell incompatible services.

(iv) Social welfare under incompatible networks is

$$W^I = 120U_A^I + 120U_B^I + \pi_A^I + \pi_B^I = 120 \cdot 40 + 120 \cdot 20 + 2400 + 4800 = 14,400.$$

Social welfare under compatible networks is

$$W^C = 120U_A^C + 120U_B^C + \pi_A^C + \pi_B^C = 120(-20) + 120(-40) + 16800 + 19200 = 28,800 > 14,400.$$

Hence, social welfare is higher when the dating services are compatible. The profit firms gain from selling compatible services dominates the reduction in consumer welfare.

Solution to Set # 2: The Components Approach

(a) (i) Incompatible systems implies that only systems X_AY_A and X_BY_B are available. Therefore, let $p_A \stackrel{\text{def}}{=} p_A^X + p_A^Y$ and $p_B \stackrel{\text{def}}{=} p_B^X + p_B^Y$.
Now, if $p_A = p_B$, consumers AB and BA are both indifferent between buying systems X_AY_A and X_BY_B, because

$$U_{ij}(X_AY_A) = \beta - p_A - \delta = \beta - p_B - \delta = U_{ij}(X_B, Y_B) \quad \text{for } i, j = A, B, \quad i \neq j.$$

This means that firm A undercuts firm B when $p_A < p_B$,
and firm B undercuts firm B when $p_B < p_A$.
Altogether, in an UPE:

I. firm A maximizes p_A subject to $\pi_B = 100p_B \geq 200p_A$, and

II. firm B maximizes p_B subject to $\pi_A = 100p_A \geq 200p_B$.

The unique solution for the two equations (under $=$) are $p_A^I = p_B^I = 0$, where superscript "I" stands for incompatible systems.

(ii)

$$\begin{aligned} \pi_A^I &= 100p_A^I = 0 \\ \pi_B^I &= 100p_B^I = 0 \end{aligned}$$

(iii) Now the 4 systems: X_AY_A, X_BY_B, X_AY_B, and X_BY_A are available for purchase. We look for an equilibrium where type AB consumers buy system X_AY_B, where as type BA consumers buy system X_BY_A.

In the market for component X, UPE implies that

$$\begin{aligned} \pi_B^X = 100p_B^X &\geq 200(p_A^X - \delta) \\ \pi_A^X = 100p_A^X &\geq 200(p_B^X - \delta) \end{aligned}$$

yielding the unique X-component prices: $p_A^X = p_B^X = 2\delta$.

In the market for component Y, UPE implies that

$$\begin{aligned} \pi_B^Y = 100p_B^Y &\geq 200(p_A^Y - \delta) \\ \pi_A^Y = 100p_A^Y &\geq 200(p_B^Y - \delta) \end{aligned}$$

yielding the unique X-component prices: $p_A^Y = p_B^Y = 2\delta$.

(iv)

$$\begin{aligned} \pi_A^C = \pi_A^X + \pi_A^Y &= 100 \cdot 2\delta + 100 \cdot 2\delta = 400\delta \\ \pi_B^C = \pi_B^X + \pi_B^Y &= 100 \cdot 2\delta + 100 \cdot 2\delta = 400\delta. \end{aligned}$$

(b) (i) Since only complete systems are sold, let p_{AA} and p_{BB} denote system prices. Then, in an UPE, the producer of AA maximizes p_{AA} subject to:

$$\pi_{BB} = 100p_{BB} \geq 200(p_{AA} - 3)$$

Similarly, the producer of BB maximizes p_{BB} subject to:

$$\pi_{AA} = 100p_{AA} \geq 200(p_{BB} - 3)$$

Solving 2 equations with 2 variables yield

$$p_{AA} = p_{BB} = 6 \quad \text{and} \quad \pi_{AA} = \pi_{BB} = 600$$

(ii) $U_{AA} = U_{BB} = 10 - 6 = 4$. Next, aggregate consumer surplus is

$$CS = 100U_{AA} + 100U_{BB} = 400 + 400 = 800.$$

Next, Social welfare is given by

$$W = CS + \pi_{AA} + \pi_{BB} = 800 + 600 + 600 = 2000.$$

(iii) In an UPE, the producer of X_A maximizes p_A^X subject to:

$$\pi_B^X = 100p_B^X \geq 200(p_A^X - 2)$$

Similarly, the producer of X_B maximizes p_B^X subject to:

$$\pi_A^X = 100p_A^X \geq 200(p_B^X - 2)$$

Solving 2 equations with 2 variables yield

$$p_A^X = p_B^X = 4 \quad \text{and} \quad \pi_A^X = \pi_B^X = 400$$

Replacing component X with Y yields:

$$p_A^Y = p_B^Y = 4 \quad \text{and} \quad \pi_A^Y = \pi_B^Y = 400$$

Therefore,

$$\pi_A = \pi_A^X + \pi_A^Y = 400 + 400 = 800 = \pi_B^X + \pi_B^Y = \pi_B.$$

(iv) The equilibrium utility level of each consumer is $U_A = U_B = 10 - 4 - 4 = 2$. Therefore,

$$CS = 100U_A + 100U_B = 400.$$

Social welfare is given by

$$W = CS + \pi_{AA} + \pi_{BB} = 400 + 800 + 800 = 2000.$$

(c) (i) Since only complete systems are sold, let p_{AA} and p_{BB} denote system prices. Then, in an UPE, where consumer AB buys system AA, and consumer BA buys system BB, the producer of AA maximizes p_{AA} subject to:

$$\pi_{BB} = 100p_{BB} \geq 200(p_{AA} - 0)$$

Similarly, the producer of BB maximizes p_{BB} subject to:

$$\pi_{AA} = 100p_{AA} \geq 200(p_{BB} - 0)$$

Solving 2 equations with 2 variables yield

$$p_{AA} = p_{BB} = 0 \quad \text{and} \quad \pi_{AA} = \pi_{BB} = 0$$

(ii) $U_{AB} = U_{BA} = 10 - 0 - 2 = 8$. Next, aggregate consumer surplus is

$$CS = 100U_{AB} + 100U_{BA} = 800 + 800 = 1600.$$

Next, Social welfare is given by

$$W = CS + \pi_{AA} + \pi_{BB} = 1600 + 0 + 0 = 1600.$$

(iii) In an UPE, the producer of X_A maximizes p_A^X subject to:

$$\pi_B^X = 100p_B^X \geq 200(p_A^X - 2)$$

Similarly, the producer of X_B maximizes p_B^X subject to:

$$\pi_A^X = 100p_A^X \geq 200(p_B^X - 2)$$

Solving 2 equations with 2 variables yield

$$p_A^X = p_B^X = 4 \quad \text{and} \quad \pi_A^X = \pi_B^X = 400$$

Replacing component X with Y yields:

$$p_A^Y = p_B^Y = 4 \quad \text{and} \quad \pi_A^Y = \pi_B^Y = 400$$

Therefore,

$$\pi_A = \pi_A^X + \pi_A^Y = 400 + 400 = 800 = \pi_B^X + \pi_B^Y = \pi_B.$$

(iv) The equilibrium utility level of each consumer is $U_A = U_B = 10 - 4 - 4 = 2$. Therefore,

$$CS = 100U_A + 100U_B = 400.$$

Social welfare is given by

$$W = CS + \pi_{AA} + \pi_{BB} = 400 + 800 + 800 = 2000.$$

(d) (i) Notice that all consumers are indifferent between the two systems when they sell of equal prices, $p_{AA} = p_{BB}$. In this case all consumers gain utility equal to $\beta - \delta$ (minus price) regardless of which system they buy. Therefore, if $p_{AA} < p_{BB}$ all consumers buy system X_AY_A, and if if $p_{AA} > p_{BB}$ all consumers buy system X_BY_B. This generates a price competition leading to marginal-cost pricing ($p_{AA} = p_{BB} = 0$ in the present case).

A formal proof for the above intuition is as follows:

$$\pi_B^I = 100p_{BB}^I = 300(p_{AA}^I - 0) \quad \text{and} \quad \pi_A^I = 200p_{AA}^I = 300(p_{BB}^I - 0)$$

yields a unique solution in which $p_{AA}^I = p_{BB}^I = 0$, where superscript "I" denotes incompatible systems. Therefore, both firms earn zero profits, $\pi_A^I = 0 \cdot q_{AA} = 0$ and $\pi_B^I = 0 \cdot q_{BB} = 0$.

(ii) In equilibrium, Firm A sells 200 units of X_A and 100 units of Y_A. Firm B sells 100 units of X_B and 200 units of Y_B.
First, we look at the market for component X. Firm A maximizes p_A^X subject to

$$\pi_B^X = 100p_B^X \geq 300(p_A^X - \delta).$$

Firm B maximizes p_B^X subject to

$$\pi_A^X = 200p_A^X \geq 300(p_B^X - \delta).$$

The UPE prices and profits (from component X only) are therefore

$$p_A^X = \frac{12\delta}{7}, \quad p_B^X = \frac{15\delta}{7}, \pi_A^X = 200p_A^X = \frac{2400\delta}{7}, \quad \text{and} \quad \pi_B^X = 100p_B^X = \frac{1500\delta}{7}.$$

Next, we explore the competition in the market for component Y. Firm A maximizes p_A^Y subject to

$$\pi_B^Y = 200p_B^Y \geq 300(p_A^Y - \delta).$$

Firm B maximizes p_B^Y subject to

$$\pi_A^Y = 100p_A^Y \geq 300(p_B^Y - \delta).$$

The UPE prices and profits (from component X only) are therefore

$$p_A^Y = \frac{15\delta}{7}, \quad p_B^Y = \frac{12\delta}{7}, \pi_A^X = 100p_A^Y = \frac{1500\delta}{7}, \quad \text{and} \quad \pi_B^Y = 200p_B^X = \frac{2400\delta}{7}.$$

Therefore, the total profit of each firm when both produce compatible components are

$$\pi_A = \pi_A^X + \pi_A^Y = \frac{3900\delta}{7} \quad \text{and} \quad \pi_B = \pi_B^X + \pi_B^Y = \frac{3900\delta}{7}.$$

Clearly, both firms earn higher profits when they produce compatible components (relative to incompatible components).

Solution to Set # 3: Software: Production and Variety

(a) The firm makes nonnegative profit as long as $TR(q) = pq \geq TC(q) = \phi + \mu q$. Hence, if

$$q \geq \frac{120000}{p - \mu} = \frac{120000}{45 - 1} = 2727.27.$$

Therefore, TaxMe™ should sell at least 2728 copies in order to make nonnegative profit.

(b) In an UPE, firm A maximizes p_A subject to:

$$\pi^I_B = 1000(p^I_B - 120) \geq 2000(p_A - 10 - 120 + s_B - s_A) = 2000(p_A - 160).$$

firm B maximizes p_B subject to:

$$\pi^I_A = 1000(p^I_B - 120) \geq 2000(p_B - 10 - 120 + s_A - s_B) = 2000(p_B - 100).$$

Solving the above two equations (under equalities) yields $p^I_A = 160$ and $p^I_B = 120$. Hence, equilibrium profits are

$$\pi^I_A = 1000(160 - 120) = 40,000 \quad \text{and} \quad \pi^I_B = 1000(120 - 120) = 0.$$

Solution to Set # 4: Software Piracy

(a) (i) Type I users pirate the software since $q - p < q$ for all $p > 0$. Hence, the number of users is at least 100.

Type O buy the software if $2(100 + 100) - p \geq 100 + 100$. Thus, to induce type O to buy the software, the price should not exceed $p = 200$.

The profit of the software publisher is $\pi^{\text{np}} = 100p = 20,000$.

(ii) Piracy is no longer an option. If only type O buy the software, the price must satisfy $p \leq 2q = 200$.

At this price, type I users will also buy the software since $q - p = 100 + 100 - p \geq 0$.

Hence, all users buy this software, so $q = 200$, hence $\pi^{\text{p}} = 200 \cdot 200 = 40,000 > 20,000$. Thus, DOORS™ should protect its software.

(b) (i) Clearly, type I users pirate the software since $q - p < q = U^I$. Therefore, if there are buyers, they must be of type O.
In order to induce type O users to buy the software, the price must be low enough to satisfy:

$$400 + q - p \geq q \quad \text{or} \quad p \leq 400.$$

Therefore, the price of software and the publisher's profit when the software is not protected are: $p^{NP} = 400$, and $\pi^{NP} = 100 \times 400 = 40,000$.

(ii) If the publisher charges as "low" price (in order to attract type$I$ to buy the software, instead of not using it at all), then the price must satisfy:

$$U^I = q - p = 100 + 200 - p \geq 0 \quad \text{or} \quad p \leq 300$$

in which case, type O users also buy the software and hence $\pi = 300 \times 300 = 90,000$. If the publisher charges a "high" price (thereby, potentially, excluding type I users) the maximum price that type O will be willing to pay must satisfy:

$$U^O = 400 + q - p = 400 + 100 - p \geq 0 \quad \text{or} \quad p \leq 500.$$

At this price, type I do not buy the software since if they buy, $U^I = 100+200-500 < 0$. Therefore, under this price the publisher earns $\pi = 100 \times 500 = 50,000$.

Clearly, this publisher maximizes profits by protecting the software and charging a price of $p^p = 300$ and earning a profit of $\pi^p = 90,000$.

(c) (i) The software is not protected. In order to induce consumers to buy the software (instead of pirate it) the price should be sufficiently low to satisfy:

$$\beta + q - p \geq q \Longrightarrow p \leq \beta$$

Hence, $p^{NP} = \beta$ and $\pi^{NP} = 100\beta$.

(ii) Since piracy is not an option, the monopoly publisher sets the highest price subject to the constraint that the consumer buys the software. That is,

$$\beta + q - p = \beta + 100 - p \geq 0 \Longrightarrow p \leq 100 + \beta$$

Hence, $p^p = 100 + \beta$, and the total profit is:

$$\pi^p = 10,000 + 100\beta - \phi = 100\beta - 12000 < \pi^{NP}$$

which means that it is not profitable to invest in protecting this software.

(d) (i) When software is unprotected, type I consumers will use the software but will not buy it. Therefore, type O will buy the software (rather than pirate it) if

$$400 + 0.5q - p \geq 0.5q - p, \quad \text{hence if} \quad p \leq 400.$$

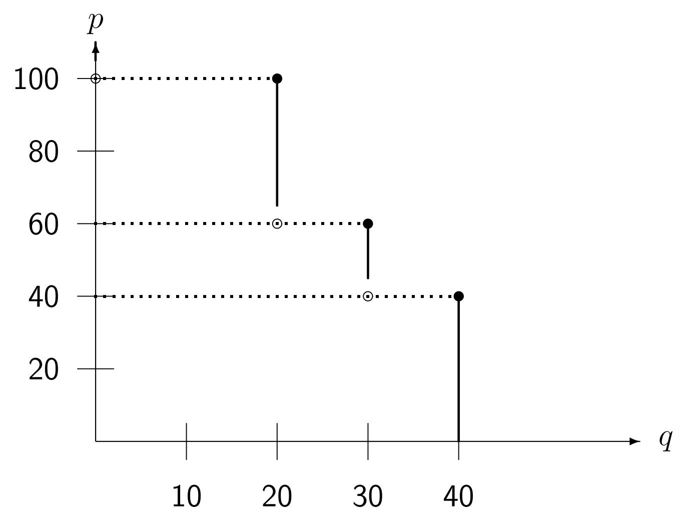
Therefore, TaxMe^{TM} sells 100 packages for a price of $p = 400$ and earns a profit of $\pi^u = 100 \cdot 400 = 40,000$.

(ii) If TaxMe^{TM} sets a low price (so all 300 consumers buy the software) it can sets $p = 150$. Notice that under this price type I consumers buy this software because $0.5 \cdot 300 - 150 \geq 0$. type O consumers will also buy this software because $400 + 0.5 \cdot 300 - 150 \geq 0$. The resulting profit is $\pi^p = 300 \cdot 150 = 45,000$.

If TaxMe^{TM} sets a high price (so only the 100 type O consumers buy the software) it sets $p = 450$. Under this price, type O consumers buy it because $400+0.5 \cdot 100-450 \geq 0$. Type I consumers don't buy it even if there are 300 users because $0.5300 - 450 < 0$. Under $p = 450$, $\pi^u = 450 \cdot 100 = 45,000$.

Hence, the seller earns a profit of $45,000$ in both cases. In this example, the seller earns a higher profit when software is protected.

Solution to Set # 5: Telecommunication



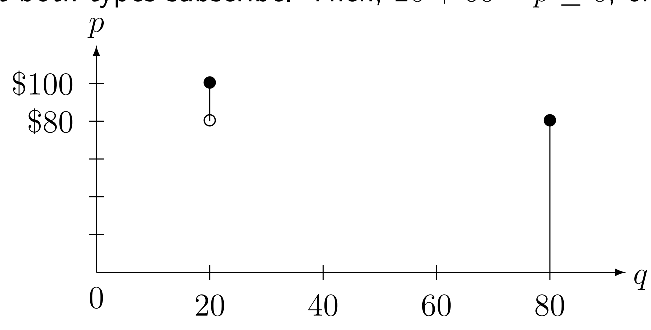
(a) (i)

(ii)

$$\begin{aligned} p = 100 \Longrightarrow q = 20 &\quad\Longrightarrow\quad \pi = (100-10)20 = 1800 \\ p = 60 \Longrightarrow q = 30 &\quad\Longrightarrow\quad \pi = (60-10)30 = 1500 \\ p = 40 \Longrightarrow q = 40 &\quad\Longrightarrow\quad \pi = (40-10)40 = 1200. \end{aligned}$$

Clearly, the profit-maximizing connection fee is $p = 100$.

(b) (i) Suppose only type H subscribe. Hence, it must be that $5 \cdot 20 - p \geq 0$, or $p \leq 100$. Now, suppose that both types subscribe. Then, $20 + 60 - p \geq 0$, or $p \leq 80$. Hence,



Formally,

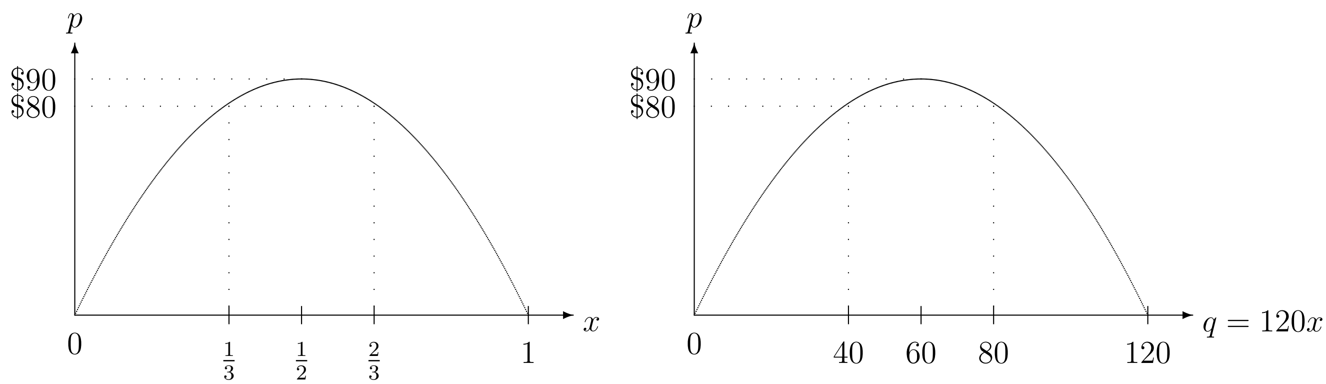
$$q(p) = \begin{cases} 0 & \text{if } p > 100 \\ 20 & \text{if } 80 < p \leq 100 \\ 20 + 60 & \text{if } p \leq 80. \end{cases}$$

(ii) In view of the above demand function, the monopoly's profit as function of the connection fee is

$$\pi(p) = \begin{cases} 0 & \text{if } p > 100 \\ (100-75)20 = \$500 & \text{if } p = 100 \\ (80-75)80 = \$400 & \text{if } p = 80. \end{cases}$$

Therefore, $p = \$100$ is the profit-maximizing price. Under this price, only 20 consumers subscribe to this service.

(c) (i) The utility function implies that the market inverse demand function (as a function of the # types subscribed) is $p = (3-3x)120x$, which is drawn below on the left.



The figure on the right "stretches" the horizontal axis by 120 to obtain price as a function of aggregate number of subscriptions (instead of the number of types). To find the maximum solve

$$0 = \frac{dp}{dx} = \frac{d[(3-3x)120x]}{dx} = 360(1-2x) \Longrightarrow x = \frac{1}{2} \Longrightarrow p = (3-3\cdot\frac{1}{2})120\cdot\frac{1}{2} = \$90.$$

(ii) To find the critical mass solve for x satisfying $80 = (3-3x)120x$ or, using a quadratic form, $360x^2 - 360x - 80 = 0$. Therefore,

$$x = \frac{360 \pm \sqrt{360^2 - 4\cdot 360\cdot 80}}{2\cdot 360} = \frac{360 \pm 120}{720} \in \left\{\frac{1}{3}\,;\,\frac{2}{3}\right\}.$$

Hence, the critical mass is

$$q^{cm} = \frac{1}{3}\,120 = 40 \text{ subscribers}.$$

(iii) A monopoly service provider maximizes profit by solving

$$\max_x \pi = p(x)120x = [(3-3x)120x]120x = 43200(x^2 - x^3).$$

The first- and second-order condition for a maximum are

$$0 = \frac{d\pi}{dx} = 43200(2x - 3x^2) \quad \text{and} \quad \frac{d^2\pi}{dx^2} = 43200(2-6x).$$

The first-order condition yields two solutions: $x = 0$ and $x = 2/3$. But,

$$\frac{d^2\pi}{dx^2}(0) = 43200(2 - 6 \cdot 0) > 0 \quad \text{and} \quad \frac{d^2\pi}{dx^2}(1/3) = 43200\left(2 - 6 \cdot \frac{2}{3}\right) < 0.$$

Hence, $x = 2/3$ is a unique maximum. The number of subscribers, price, and profit are

$$q = 120 \cdot \frac{2}{3} = 80, \quad p = \left(3 - 3 \cdot \frac{2}{3}\right) 120 \cdot \frac{2}{3} = \$80, \quad \text{and} \quad \pi = p \cdot q = 80 \cdot 80 = \$6400.$$

(d) The total traffic on the incumbent's local network is $Q = q_L^I + d^I + d^E = 600,000$ phone calls. The access fee under the fully distributed costs rule is

$$a = \mu_L^I + \frac{\phi}{Q} = 0.5 + \frac{1,200,000}{600,000} = \frac{5}{2} = 2.5.$$

That is, the entrant pays the incumbent's marginal cost of terminating a LD call on the local network plus the share in the maintenance cost of the local network.

The access fee under the ECPR is

$$a = p^I - \mu^I = 4 - 3 = 1.$$

Under this rule, the entrant "compensates" for the loss of having consumers placing the phone call on the entrant's LD network rather than on the incumbent's LD network.

(e) (i)

$$\begin{aligned} \pi_N &= (a - a)\eta_N + a\eta_S = a\eta_S \Longrightarrow a_N = \beta \\ \pi_S &= (\beta - a)\eta_S + a\eta_N = \beta\eta_S + a(\eta_N - \eta_S) \Longrightarrow a_S = \beta \end{aligned}$$

(ii)

$$\hat{a} = \frac{\beta + \beta}{2} = \beta \Longrightarrow T_{\vec{NS}} = \beta\eta_N - \beta\eta_S = \beta(\eta_N - \eta_S) > 0.$$

(iii)

$$\begin{aligned} \pi_N &= (a - a)\eta_N + a\eta_S = a\eta_S \Longrightarrow a_N = \beta \\ \pi_S &= (a - a)\eta_S + a\eta_N = a\eta_N \Longrightarrow a_S = \beta \end{aligned}$$

Hence,

$$\hat{a} = (\beta + \beta)/2 = \beta \Longrightarrow T_{\vec{NS}} = \beta\eta_N - \beta\eta_S = \beta(\eta_N - \eta_S) > 0.$$

Solution to Set # 6: Broadcasting

(a) (i) There are 6 Nash equilibria in broadcasting time given by

$$\begin{aligned}\langle t_A, t_B, t_C\rangle &= \langle 17, 17, 18\rangle\\ &\langle 17, 18, 17\rangle\\ &\langle 18, 17, 17\rangle\\ &\langle 18, 18, 17\rangle\\ &\langle 18, 17, 18\rangle\\ &\langle 17, 18, 18\rangle.\end{aligned}$$

(ii) The unique Nash equilibrium is $\langle t_A, t_B, t_C\rangle = \langle 17, 17, 17\rangle$.

Proof.
In equilibrium, all viewers are equally distributed among the 3 stations, so

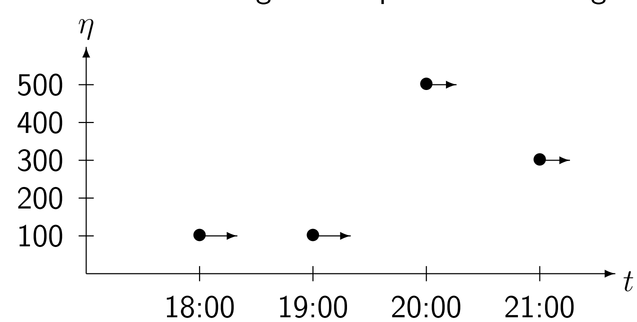
$$\pi_A = \pi_B = \pi_C = \frac{4\eta}{3}.$$

Suppose that station A deviates to $\tilde{t}_A = 18$. Then, only η viewers will watch station A, hence

$$\tilde{\pi}_A = \eta < \frac{4\eta}{3}.$$

Therefore, no station will unilaterally deviate to broadcast at $t = 18$.

(b) The distribution of viewers' ideal watching time is plotted on the figure below.



The arrows indicate that viewers can postpone their watching but are unable advance it due to work obligations. The broadcasting time for network i as a function of broadcasting time of network j is

$$t_i = BR_i(t_j) = \begin{cases} 21 & \text{if } t_j = 18 \text{ (hence, } \pi_i = 900\rho) \\ 21 & \text{if } t_j = 19 \text{ (hence, } \pi_i = 800\rho) \\ 20 & \text{if } t_j = 20 \text{ (hence, } \pi_i = 350\rho) \\ 20 & \text{if } t_j = 21 \text{ (hence, } \pi_i = 700\rho). \end{cases}$$

The unique Nash equilibrium is $\langle t_A, t_B \rangle = \langle 20, 20 \rangle$. The networks split the $100 + 100 + 500$ viewers whose ideal watching time are: 18:00, 19:00, and 20:00 (other viewers cannot watch). Therefore, $\pi_A(20, 20) = \pi_B(20, 20) = 350\rho$.

(c) (i) The outcome $\langle p_A, p_B, p_C, p_D \rangle = \langle T, T, T, N \rangle$ is a NE. *Proof*:

$$\begin{aligned}
\pi_A(T,T,T,N) = 800/3 &> 200 = \pi_A(N,T,T,N) \\
\pi_A(T,T,T,N) = 800/3 &> 200 = \pi_A(M,T,T,N) \\
\pi_B(T,T,T,N) = 800/3 &> 200 = \pi_B(T,N,T,N) \\
\pi_B(T,T,T,N) = 800/3 &> 200 = \pi_B(T,M,T,N) \\
\pi_C(T,T,T,N) = 800/3 &> 200 = \pi_C(T,T,N,N) \\
\pi_C(T,T,T,N) = 800/3 &> 200 = \pi_C(T,T,M,N) \\
\pi_D(T,T,T,N) = 400 &> 200 = \pi_D(T,T,T,M) \\
\pi_D(T,T,T,N) = 400 &> 200 = \pi_D(T,T,T,T)
\end{aligned}$$

(ii) In the above NE,

$$CS = 800U_T + 400U_N + 200U_M = 800 \times 5 + 400 \times 5 + 200 \times 0 = 6000.$$

Total industry profit is:

$$T\pi = \pi_A + \pi_B + \pi_C + \pi_D = \frac{800}{3} + \frac{800}{3} + \frac{800}{3} + 400 = 1200.$$

Finally, the equilibrium level of social welfare is:

$$W(T,T,T,N) = CS + T\pi = 6000 + 1200 = 7200.$$

We now prove that the outcome $\langle T, T, T, N \rangle$ does not maximize social welfare. Consider now the outcome $\langle T, T, M, N \rangle$.

$$CS = 800U_T + 400U_N + 200U_M = 800 \times 5 + 400 \times 5 + 200 \times 5 = 7000.$$

Also,

$$T\pi = \pi_A + \pi_B + \pi_C + \pi_D = \frac{800}{2} + \frac{800}{2} + 200 + 400 = 1400.$$

Therefore,

$$W(T,T,M,N) = CS + T\pi = 7000 + 1400 = 8400 > 7200 = W(T,T,T,N).$$

(d) (i) $\langle p_A, p_B, p_C\rangle = \langle 1, 1, 2\rangle$ is a NE (among several others). Proof:

$$\begin{aligned}
\pi_A(1,1,2) = 200 &\geq 75 = \pi_A(2,1,2)\\
\pi_A(1,1,2) = 200 &\geq 100 = \pi_A(3,1,2)\\
\pi_A(1,1,2) = 200 &\geq 80 = \pi_A(4,1,2)\\
\pi_B(1,1,2) = 200 &\geq 75 = \pi_B(1,2,2)\\
\pi_B(1,1,2) = 200 &\geq 100 = \pi_B(1,3,2)\\
\pi_B(1,1,2) = 200 &\geq 80 = \pi_B(1,4,2)\\
\pi_C(1,1,2) = 150 &\geq 133 = \pi_C(1,1,1)\\
\pi_C(1,1,2) = 150 &\geq 100 = \pi_C(1,1,3)\\
\pi_C(1,1,2) = 150 &\geq 80 = \pi_C(1,1,4)
\end{aligned}$$

(ii) The sum of viewers' utilities is maximized when

$$\langle p_A, p_B, p_C\rangle = \langle 1, 2, 3\rangle$$

in which case

$$W(1,2,3) = 400\beta + 150\beta + 100\beta + 80 \times 0 = 650\beta + 650\rho.$$

Remark: You can observe the market failure since at the NE of the previous section,

$$W(1,1,2) = 400\beta + 150\beta + 100 \times 0 + 80 \times 0 = 550\beta + 550\rho < 650\beta + 650\rho.$$

(e) (i)

$$\begin{aligned}
p_C = 5 &\implies q_C = 9000 \implies \pi_C = 9000 \times 5 = 45,000\\
p_C = 4 &\implies q_C = 10000 \implies \pi_C = 10000 \times 4 = 40,000\\
p_C = 1 &\implies q_C = 11000 \implies \pi_C = 11000 \times 1 = 11,000
\end{aligned}$$

Therefore, the profit-maximizing price for CNN is $p_C = 5$ resulting in a profit of $\pi_C = 45,000$.
Similarly, the profit-maximizing price for BBC is $p_B = 5$ resulting in a profit of $\pi_B = 45,000$.
Altogether, the profit of this Cable-TV provider is $\pi = \pi_C + \pi_B = 90,000$.

(ii)

$$\begin{aligned}
p_{CB} = 5 &\implies q_{CB} = 11000 \implies \pi_{CB} = 11000 \times 5 = 55,000\\
p_{CB} = 10 &\implies q_{CB} = 9000 \implies \pi_{CB} = 9000 \times 10 = 90,000
\end{aligned}$$

Therefore, the profit-maximizing price of the package is $p_{CB} = 5$ and the associated profit level is $\pi_{CB} = 90,000$. Notice that in this case pure tying in not profit-increasing.

Solution to Set # 7: Markets for Information

(a) (i) The publisher's profit-maximizing price is $p^b = 23$. Hence, the profit when selling to individual readers only is $\pi^b = (23 - 12)1200 = 13,200$.

(ii) The maximum rental price each library i can charge each reader is $p_i^r = 23/2$. Therefore, the maximum price in which each library is willing to pay for one copy of the book is:

$$p_i = \left(\frac{\eta}{\lambda}\right) p_i^r = \left(\frac{1200}{50}\right)\left(\frac{23}{2}\right) = 276.$$

Now, the publisher produces 50 copies (one for each library), and therefore earns a profit of:

$$\pi = (p_i - \mu)50 = (276 - 12)50 = 13,200.$$

(b) (i) Each resident takes the demand by all other residents q_j for $i \neq j$ as given and chooses her Internet usage level q_i to maximize the above utility. The first-order condition is

$$0 = \frac{dU_i}{dq_i} = \frac{1}{2\sqrt{q_i}} - \left(\frac{1}{2}\right)\left(\frac{1}{96}\right) - p.$$

Because, Internet is free, setting $p = 0$ yields $q_i = 96^2 = 9216$ which is the equilibrium demand by each resident in Mbps. Hence, aggregate demand is $Q = \eta q_i = 12 \cdot 9216 = 110,592$ Mbps. Clearly, the network is heavily congested because $Q = 110,592 > 96 = \bar{Q}$.

(ii) The social planner chooses a uniform consumption level q by each resident to maximize social welfare given by

$$\max_q W = 12U_i + \pi^{\mathsf{ISP}} = 12\left[\sqrt{q} - \left(\frac{1}{2}\right)\left(\frac{Q}{96}\right) - pq\right] + 12pq = 12\left[\sqrt{q} - \left(\frac{1}{2}\right)\left(\frac{12q}{96}\right)\right].$$

Clearly, total consumer expenditure cancels out with total profit of the ISP. Also, note that $Q = 12q$. The first-order condition for a maximum is

$$0 = \frac{dW}{dq} = 12\left[\frac{1}{2\sqrt{q}} - \left(\frac{1}{2}\right)\left(\frac{12}{96}\right)\right] \Longrightarrow q^* = 64 \quad \text{and} \quad Q^* = 12q = 768.$$

The network is still congested because $Q^* = 768 > 96$, but much less congested compared with the equilibrium level.

(iii) To find the price which would induce all resident to consume at the socially-optimal level note from part (a) that residents choose their consumption level q_i to satisfy

$$p = \frac{1}{2\sqrt{q_i}} - \left(\frac{1}{2}\right)\left(\frac{1}{96}\right) = \frac{1}{2\sqrt{64}} - \left(\frac{1}{2}\right)\left(\frac{1}{96}\right) = \frac{11}{192} \approx 0.057.$$

Solution to Set # 8: Switching Costs

(a) An UPE is the solution for

I. firm A maximizes p_A subject to $\pi_B = 200p_B \geq 300(p_A - \delta + 300 - 100)$, yielding $\delta = 299$, and

II. firm B maximizes p_B subject to $\pi_A = 100p_A \geq 300(p_B - \delta + 300 - 200)$ yielding $\delta = 199$.

(b) In an UPE, bank A maximizes f_A subject to:

$$\pi_B = 200f_B \geq (200 + 100)(f_A - \delta_A)$$

Hence,

$$200 \times 30 = 300(30 - \delta_A) \Longrightarrow \delta_A = 10.$$

In an UPE, bank B maximizes f_B subject to:

$$\pi_A = 100f_A \geq (100 + 200)(f_B - \delta_B)$$

Hence,

$$100 \times 30 = 300(30 - \delta_B) \Longrightarrow \delta_B = 20.$$

(c) Bank A (the largest) maximizes its fee f_A subject to

$$\pi_C = 200 \cdot 60 \geq (200 + 600)(60 - \delta_A) \Longrightarrow \delta_A = \$45.$$

Bank B maximizes its fee f_B subject to

$$\pi_C = 200 \cdot 60 \geq (200 + 400)(60 - \delta_B) \Longrightarrow \delta_B = \$40.$$

Bank C (the smallest) maximizes its fee f_C subject to

$$\pi_A = 600 \cdot 60 \geq (200 + 600)(60 - \delta_C) \Longrightarrow \delta_C = \$15.$$

Solution to Set # 9: The Airline Industry

(a) (i) Suppose $\eta_1 = \eta_2 = \eta_3 = \eta > 0$. Calculate which network of operation (FC or HS) minimized the airline's cost. Show your calculation!

Fully-connected network: $TC^{\mathsf{FC}} = 3000 + 3\sqrt{\eta}$

Hub-and-spoke network: $TC^{\mathsf{HS}} = 2000 + 2\sqrt{2\eta}$

Clearly, $TC^{\mathsf{FC}} > TC^{\mathsf{HS}}$ since even without taking fixed costs into account, $3\sqrt{\eta} > 2\sqrt{2\eta} \approx 2.82\sqrt{\eta}$.

(ii) Denote by TC_{city} the airline's total cost when the hub is located in this city. Then,

$$\begin{aligned}
TC_A &= 2000 + \sqrt{100+800} + \sqrt{800+800} = 2000 + 30 + 40 = 2070 \\
TC_B &= 2000 + \sqrt{100+800} + \sqrt{800+800} = 2000 + 30 + 40 = 2070 \\
TC_C &= 2000 + \sqrt{100+800} + \sqrt{100+800} = 2000 + 30 + 30 = 2060.
\end{aligned}$$

Clearly, the hub should be located in city C.

(b) (i) Since route 1 and route 2 passengers fly directly, a monopoly airline would set $p_1 = p_2 = 12$. To set the profit-maximizing p_3, two cases must be analyzed, First, $p_3 = 8$ in which case $q_3 = 50 + 10$ and hence,

$$\Pi(p_3 = 8) = 2(50+10)12 + (50+10)8 - 2 \cdot 200 = 1520.$$

The second option is to set a high price, $p_3 = 12$, so $q_3 = 10$. In this case,

$$\Pi(p_3 = 12) = 2(50+10)12 + 10 \cdot 12 - 2 \cdot 200 = 1160.$$

Therefore, under the HS network, the monopoly airline sets $p_1 = p_2 = 12$ and $p_3 = 8$, and earns $\Pi^{HS} = 1520$.

(ii) Under FC network, all passengers fly directly to destination, hence the monopoly airline can charge $p_i = 12$. Total profit is

$$\Pi^{FC} = \pi_1 + \pi_2 + \pi_3 = 3(50+10)12 - 3 \cdot 200 = 1560.$$

Comparing (7a) with (7b) reveals that the FC is the most profitable network of operation.

(c) (i) Airline α maximizes the airfare p_α subject to:

$$\pi_\beta = \eta p_\beta \geq 2\eta(p_\alpha - 4 + f_\beta - f_\alpha) = 2\eta(p_\alpha - 4 + 3 - 6) = 2\eta(p_\alpha - 7).$$

Airline β maximizes the airfare p_β subject to:

$$\pi_\alpha = \eta p_\alpha \geq 2\eta(p_\beta - 4 + f_\alpha - f_\beta) = 2\eta(p_\beta - 4 + 6 - 3) = 2\eta(p_\beta - 1).$$

Solving 2 equations with 2 variables, the equilibrium airfares and airlines' profit levels are given by

$$p_\alpha = 10, \quad p_\beta = 6, \quad \text{and} \quad \pi_\alpha = 10\eta, \quad \pi_\beta = 6\eta.$$

(ii) Under a code-sharing agreement, passengers of all airlines are exposed to the same frequency of flights given by $f = f_\alpha + f_\beta$.
Airline α maximizes the airfare p_α subject to:

$$\pi_\beta = \eta p_\beta \geq 2\eta(p_\alpha - 4 + f - f) = 2\eta(p_\alpha - 4).$$

Airline β maximizes the airfare p_β subject to:

$$\pi_\alpha = \eta p_\alpha \geq 2\eta(p_\beta - 4 + f - f) = 2\eta(p_\beta - 4).$$

Solving 2 equations with 2 variables, the equilibrium airfares and airlines' profit levels are given by

$$p_\alpha = 8, \quad \text{and} \quad \pi_\alpha = \pi_\beta = 8\eta.$$

Clearly airline β benefits from the code-sharing agreement since it provides a lower frequency of flights. Airline α loses from this agreement since it can no longer charge an extra premium for providing a higher frequency of flights.

Solution to Set # 10: Languages as Networks

(a) (i) $\langle n_{ES}, n_{SE}\rangle = \langle 60, 0\rangle$ is an equilibrium since:

$$(n_{SE} = 0) \Longrightarrow \frac{60+40}{10} - 3 > \frac{60}{10},$$

meaning that all E-speakers are better off learning S. In addition,

$$(n_{ES} = 60) \Longrightarrow \frac{40+60}{10} - 7 < \frac{40+60}{10},$$

meaning that given that all E-speakers learn S, there it is not beneficial for S-speakers to learn E.

Finally, $\langle n_{ES}, n_{SE}\rangle = \langle 0, 40\rangle$ is *not* an equilibrium since

$$(n_{ES} = 0) \Longrightarrow \frac{40+60}{10} - 7 < \frac{40}{10},$$

meaning that all S-speakers do not benefit from learning E even if E-speakers do not learn S.

(ii) We define social welfare as the sum of utilities. Formally, let $W \stackrel{\text{def}}{=} 60U_A + 40U_B$. Then,

$$\begin{aligned} W(60,0) &= 60\left(\frac{100}{10}-3\right)+40\left(\frac{100}{10}\right)=820 \\ W(0,40) &= 60\left(\frac{100}{10}\right)+40\left(\frac{100}{10}-7\right)=720 \\ W(60,40) &= 60\left(\frac{100}{10}-3\right)+40\left(\frac{100}{10}-7\right)=540 \\ W(0,0) &= 60\left(\frac{60}{10}\right)+40\left(\frac{40}{10}\right)=520. \end{aligned}$$

Clearly, $\langle n_{ES}, n_{SE}\rangle = \langle 60, 0\rangle$ is the socially optimal outcome.

(b) (i) $\langle n_{ES}, n_{SE}\rangle = \langle 0, 0\rangle$ (i.e., no one learns any language) is an equilibrium. *Proof*: Given $n_{SE}=0$,

$$U_E(0,0)=\frac{60+0}{10}=6>5=\frac{60+40}{10}-5=U_E(60,0)$$

hence, not learning Spanish yields a higher utility to English native speakers. Given $n_{ES}=0$,

$$U_S(0,0)=\frac{40+0}{10}=4>3=\frac{40+60}{10}-7=U_S(0,40)$$

hence, not learning English yields a higher utility to Spanish native speakers.

(ii) Social welfare is defined by: $W = 60U_E + 40U_S$. Now,

$$\begin{aligned} W(0,0) &= 60\frac{60}{10}+40\frac{40}{10}=520 \\ W(60,0) &= 60\left(\frac{60+40}{10}-5\right)+40\frac{40+60}{10}=700 \\ W(0,40) &= 60\frac{100}{10}+40\left(\frac{100}{10}-7\right)=720 \\ W(60,40) &= 60\left(\frac{100}{10}-5\right)+40\left(\frac{100}{10}-7\right)=420 \end{aligned}$$

Therefore, the socially-optimal learning outcome is $\langle n_{ES}, n_{SE}\rangle = \langle 0, 40\rangle$ meaning that only the native Spanish speaking learn English.

(c) (i) Yes, $n_{HE}=40$ and $n_{BE}=60$ is a language acquisition equilibrium. To prove, we show that no group of speakers can benefit from deviating from $n_{HE}=40$ and $n_{BE}=60$. Given $n_{HE}=40$ and $n_{BE}=60$, the utility of a Bengali native speaker is

$$U_B = \begin{cases} 60 & \text{does not study} \\ 100-30=70 & \text{studies Hindi} \\ 60+0+40-30=70 & \text{studies English} \end{cases}$$

Hence, Bengali speakers don't have an incentive to deviate and they will all study English, $n_{BE} = 60$. Similarly, the utility of an Hindi native speaker is

$$U_H = \begin{cases} 40 & \text{does not study} \\ 100 - 30 = 70 & \text{studies Bengali} \\ 40 + 0 + 60 - 30 = 70 & \text{studies English} \end{cases}$$

Hence, Hindi speakers don't have an incentive to deviate and they will all study English, $n_{HE} = 40$.

(ii) I. $CS^I = 60(40 + 0 + 60 - 30) + 40(60 + 0 + 40 - 30) = 7000$.

II. $CS^{II} = 60(100 - 30) + 40(40 + 60) = 8200$.

III. $CS^{III} = 60(60 + 40) + 40(100 - 30) = 8800$.

THE END

