General Instructions for Students

1. The problem sets given in this handout are taken from old exams. *Note*: The questions given in the Winter–2009 semester midterm and final exams are still not incorporated in this version. You can download these exams and solutions from `www.ozshy.com`.
2. Exercises should NOT submitted (they will not be graded). However,
3. The best, and perhaps the only, way to ensure that you understand the material taught in class is to solve these exercises under "exam conditions" and only then check the proposed solution.
4. Solutions to all problems can be downloaded as a separate file.
5. Another advantage of solving these exercises is that they provide the best preparation for the exams. Most exam questions will be based on variations of these exercises.

Set # 1: The Network Externalities Approach

(a) Consider a duopoly (two-firm) computer industry producing two brands named *Artichoke* (Brand A), and *Banana* (Brand B). Assume that each computer costs \$2 to produce. Let p_A denote the price charged by Artichoke, and p_B the price charged by Banana.

There are 100 consumers who are brand A-oriented consumers, and 100 consumers who are brand B-oriented consumers. Let q_A be the number of consumers who purchase brand A, and q_B the number of consumers who purchase brand B. Formally, the utility of A-oriented and B-oriented consumers are given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} 0.5q_A - p_A & \text{buys } A \text{ ; } A \text{ is incompatible} \\ 0.5q_B - p_B - 300 & \text{buys } B \text{ ; } B \text{ is incompatible} \\ 0.5(q_A + q_B) - p_A & \text{buys } A \text{ ; } A \text{ is } B\text{-compatible} \\ 0.5(q_A + q_B) - p_B - 300 & \text{buys } B \text{ ; } B \text{ is } A\text{-compatible,} \end{cases} \tag{1}$$

and

$$U_B \stackrel{\text{def}}{=} \begin{cases} 0.5q_A - p_A - 300 & \text{buys } A \text{ ; } A \text{ is incompatible} \\ 0.5q_B - p_B & \text{buys } B \text{ ; } B \text{ is incompatible} \\ 0.5(q_A + q_B) - p_A - 300 & \text{buys } A \text{ ; } A \text{ is } B\text{-compatible} \\ 0.5(q_A + q_B) - p_B & \text{buys } B \text{ ; } B \text{ is } A\text{-compatible.} \end{cases}$$

Solve the following problems:

(i) Calculate the undercut-proof equilibrium prices assuming that the computer brands are <u>incompatible</u>. *Hint:* First make sure that you know to define price-undercutting considering the fact that each unit costs \$2 to produce.

(ii) Calculate the equilibrium profit level of each firm when the brands are incompatible.

(iii) Calculate the undercut-proof equilibrium prices assuming that the computer brands are compatible.

(iv) Calculate the equilibrium profit level of each firm when the brands are compatible.

(b) The online dating industry consists of two online service providers labeled as A and B. Assume production is costless ($\mu_A = \mu_B = 0$). Let p_A denote the price charged by provider A, and p_B the price charged by provider B.

There are 120 consumers who are A-oriented, and 120 consumers who are B-oriented. Let q_A be the number of consumers who subscribe to A, and q_B the number of consumers who subscribe to B. Formally, the utility of an A-oriented and B-oriented consumer is given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} \frac{1}{2}q_A - p_A & \text{subscribes to } A \\ \frac{1}{2}q_B - p_B - 60 & \text{subscribes to } B \end{cases} \quad \text{and} \quad U_B \stackrel{\text{def}}{=} \begin{cases} \frac{1}{2}q_A - p_A - 90 & \text{subscribes to } A \\ \frac{1}{2}q_B - p_B & \text{subscribes to } B \end{cases}$$

Notice that this model describes an online service industry in which A-oriented subscribers find it easier to switch from provider A to provider B, than for B-oriented to switch from B to A.

(i) Compute the UPE prices and the profit of each provider assuming that the two online providers are *incompatible* in the sense that A subscribers can date only other A subscribers and B subscribers can date only other B subscribers.

(ii) Compute the UPE prices and profit level of each provider assuming that the two dating online providers are *compatible* in the sense that A and B subscribers can date all other A and B subscribers.

(iii) Compute the utility levels of A and B subscribers when the dating services are incompatible and compatible. Which subscribers prefer compatible dating services and which are better off when these services are incompatible? Provide some intuition for your result.

(iv) Compute and compare social welfare levels under incompatible and compatible online dating services.

Set # 2: The Components Approach

(a) Consider a system composed of two components labeled X and Y. There are two firms producing two different systems (different brands), at zero production cost. Firm A produces components X_A and Y_A, and firm B produces X_B and Y_B. In this market there are 100 consumers labeled AB, and 100 consumers labeled BA. The Utility function of a consumer i, j where $i, j = A, B$ is

$$U_{i,j} = \begin{cases} \beta - \left(p_i^X + p_j^Y\right) & \text{buys system } X_iY_j \\ \beta - \left(p_j^X + p_j^Y\right) - \delta & \text{buys system } X_jY_j \\ \beta - \left(p_i^X + p_i^Y\right) - \delta & \text{buys system } X_iY_i \\ \beta - \left(p_j^X + p_i^Y\right) - 2\delta & \text{buys system } X_jY_i \end{cases}$$

Solve the following problems:

(i) Calculate the undercut-proof equilibrium prices assuming that the components produced by different firms are incompatible. *Hint:* First make sure that you know to define price-undercutting.

(ii) Calculate the equilibrium profit level of each firm.

(iii) Calculate the undercut-proof equilibrium prices assuming that the components produced by different firms are compatible.

(iv) Calculate the equilibrium profit level of each firm.

(b) Consider a system composed of two components labeled X and Y. There are two firms producing two different systems (different brands), at zero production cost. Firm A produces components X_A and Y_A, and firm B produces X_B and Y_B. In this market there are 100 consumers labeled AA, and 100 consumers labeled BB. The Utility function of a consumer i, j where $i, j = A, B$ is

$$U_{i,j} = \begin{cases} 10 - \left(p_i^X + p_j^Y\right) & \text{buys system } X_iY_j \\ 10 - \left(p_j^X + p_j^Y\right) - 2 & \text{buys system } X_jY_j \\ 10 - \left(p_i^X + p_i^Y\right) - 2 & \text{buys system } X_iY_i \\ 10 - \left(p_j^X + p_i^Y\right) - 3 & \text{buys system } X_jY_i \end{cases}$$

Solve the following problems:

(i) Calculate the undercut-proof equilibrium prices and the profit of each firm assuming that the components produced by different firms are incompatible. *Hint:* First make sure that you know to define price-undercutting.

(ii) Calculate the aggregate consumer surplus and social welfare.

(iii) Calculate the undercut-proof equilibrium prices and firms' profit levels assuming that the components produced by different firms are compatible.

(iv) Calculate the aggregate consumer surplus and social welfare.

(c) Consider a system composed of two components labeled X and Y. There are two firms producing two different systems (different brands), at zero production cost. Firm A produces components X_A and Y_A, and firm B produces X_B and Y_B. In this market there are 100 consumers labeled AB, and 100 consumers labeled BA. The Utility function of a consumer i, j where $i, j = A, B$ is

$$U_{i,j} = \begin{cases} 10 - \left(p_i^X + p_j^Y\right) & \text{buys system } X_i Y_j \\ 10 - \left(p_j^X + p_j^Y\right) - 2 & \text{buys system } X_j Y_j \\ 10 - \left(p_i^X + p_i^Y\right) - 2 & \text{buys system } X_i Y_i \\ 10 - \left(p_j^X + p_i^Y\right) - 4 & \text{buys system } X_j Y_i \end{cases}$$

(i) Calculate the undercut-proof equilibrium prices and the profit of each firm assuming that the components produced by different firms are incompatible. *Hint:* First make sure that you know to define price-undercutting.

(ii) Calculate the aggregate consumer surplus and social welfare.

(iii) Calculate the undercut-proof equilibrium prices and the firms' profit levels assuming that the components produced by different firms are compatible.

(iv) Calculate the aggregate consumer surplus and social welfare.

(d) Consider a system composed of two components labeled X and Y. There are two firms producing two different systems (different brands), at zero production cost. Firm A produces components X_A and Y_A, and firm B produces X_B and Y_B. In this market there are 200 (two-hundred) consumers labeled AB, and 100 (one-hundred) consumers labeled BA. The Utility function of a consumer (i, j) is

$$U_{i,j} = \begin{cases} \beta - \left(p_i^X + p_j^Y\right) & \text{buys system } X_i Y_j \\ \beta - \left(p_j^X + p_j^Y\right) - \delta & \text{buys system } X_j Y_j \\ \beta - \left(p_i^X + p_i^Y\right) - \delta & \text{buys system } X_i Y_i \\ \beta - \left(p_j^X + p_i^Y\right) - 2\delta & \text{buys system } X_j Y_i, \end{cases} \quad \text{where} \quad i, j = A, B.$$

(i) Let p_{AA} denote the price of system $X_A Y_A$, and p_{BB} denote the price of system $X_B Y_B$. Assuming that firms produce *incompatible* components, compute the Undercut-Proof equilibrium system prices and the equilibrium profit levels π_A and π_B.

(ii) Compute the component prices in an Undercut-proof equilibrium assuming that the firms produce compatible components. Compute the profit levels and compare them to the profit levels when the firms produce incompatible components.

Set # 3: Software Production and Software Variety

(a) Consider the TAXME™ company which is a leader in tax preparation software. Suppose the software costs $\phi = \$120,000$ to develop, and in addition $\mu = \$1$ to duplicate and sell each copy.

Assuming that TAXME™ sells each software package for $p = \$45$, compute the minimum number of copies that must be sold in order for TAXME™ to make positive profit.

(b) Consider the market for computers with two brand producing firms labeled as A and B. Assume that each unit of hardware costs 120 to produce, so $c_A = c_B = 120$. Let p_A and p_B denote the prices of hardware A and B, respectively. Suppose that there are $s_A = 90$ software packages written specifically for hardware A, and $s_B = 60$ specifically for hardware B. There are $\eta_A = 1000$ A-oriented consumers and $\eta_B = 1000$ B-oriented consumers whose utility functions are given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} s_A - p_A & \text{buys } A \text{ ; } A \text{ is incompatible} \\ s_B - p_B - 10 & \text{buys } B \text{ ; } B \text{ is incompatible} \\ s_A + s_B - p_A & \text{buys } A \text{ ; } A \text{ is } B\text{-compatible} \\ s_A + s_B - p_B - 10 & \text{buys } B \text{ ; } B \text{ is } A\text{-compatible,} \end{cases} \tag{2}$$

and

$$U_B \stackrel{\text{def}}{=} \begin{cases} s_A - p_A - 10 & \text{buys } A \text{ ; } A \text{ is incompatible} \\ s_B - p_B & \text{buys } B \text{ ; } B \text{ is incompatible} \\ s_A + s_B - p_A - 10 & \text{buys } A \text{ ; } A \text{ is } B\text{-compatible} \\ s_A + s_B - p_B & \text{buys } B \text{ ; } B \text{ is } A\text{-compatible.} \end{cases}$$

Compute the UPE hardware prices, p_A^I and p_B^I, and the firms' profits, π_A^I and π_B^I, assuming that the two hardware are *incompatible*, in the sense that hardware A can run only A-specific software whereas hardware B can run only B-specific software.

Set # 4: Software Piracy

(a) Consider a market for a popular software DOORS™. There are 100 support-oriented (type-O) consumers, and 100 support-independent (type-I) consumers, with utility functions given by

$$U^O \stackrel{\text{def}}{=} \begin{cases} 2q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases} \quad U^I \stackrel{\text{def}}{=} \begin{cases} q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases}$$

where q denotes the number of users of this software (which includes the number of buyers and the number of pirates, if piracy takes place). Suppose that the software is costless to produce and costless to protect. Also, assume that DOORS™ provides support only to those consumers who buy the software.

Solve the following problems:

(i) Suppose that DOORS™ is *not* protected, so piracy is an option for every consumer. Calculate the software seller's profit-maximizing price. Prove your answer.

(ii) Suppose that $\textsc{Doors}^{\text{TM}}$ is protected, so piracy is impossible. Calculate the software seller's profit-maximizing price. Prove your answer.

(b) Consider a market for a popular software $\textsc{Acropop}^{\text{TM}}$. There are 100 (one-hundred) support-oriented (type-O) users, and 200 (two-hundred) support-independent (type-I) users, with utility functions given by

$$U^O \stackrel{\text{def}}{=} \begin{cases} 400 + q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases} \quad U^I \stackrel{\text{def}}{=} \begin{cases} q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases}$$

where q denotes the number of users of this software (which includes the number of buyers and the number of pirates, if piracy takes place). Suppose that the software is costless to produce and costless to protect. Also, assume that $\textsc{Acropop}^{\text{TM}}$ provides support only to those consumers who buy the software.

Solve the following problems:

(i) Suppose that $\textsc{Acropop}^{\text{TM}}$ is *not* protected, so piracy is an option for every consumer. Calculate the software seller's profit-maximizing price. Prove your answer.

(ii) Suppose that $\textsc{Acropop}^{\text{TM}}$ is protected, so piracy is impossible. Calculate the software seller's profit-maximizing price. Prove your answer.

(c) Consider a market for a popular software $\textsc{Acropop}^{\text{TM}}$. There are 100 (one-hundred) identical users, each with a utility function given by

$$U \stackrel{\text{def}}{=} \begin{cases} \beta + q - p & \text{buys the software} \\ q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases}$$

where $\beta > 0$ measures the value of service provided by the software firm to its buyers, and q denotes the number of users of this software (which includes the number of buyers and the number of pirates, if piracy takes place). Suppose that the software is costless to produce. Also, assume that $\textsc{Acropop}^{\text{TM}}$ provides support only to those consumers who buy the software.

Solve the following problems:

(i) Suppose that $\textsc{Acropop}^{\text{TM}}$ is *not* protected, so piracy is an option for every consumer. Calculate the software seller's profit-maximizing price. Prove your answer.

(ii) Suppose now that $\textsc{Acropop}^{\text{TM}}$ can invest a fixed (one time) amount of $\phi = 12,000$ to protect against piracy, so piracy becomes impossible. Calculate the software seller's profit-maximizing price and the profit level if the publisher invests in this anti-piracy measure. Prove your answer.

(d) Consider a market for a popular tax preparation software TaxMe™. There are 100 (one-hundred) support-oriented (type-O) users, and 200 (two-hundred) support-independent (type-I) users, with utility functions given by

$$U^O \stackrel{\text{def}}{=} \begin{cases} 400 + 0.5q - p & \text{buys the software} \\ 0.5q & \text{pirates (steals) the software} \\ 0 & \text{does not use this software,} \end{cases} \quad U^I \stackrel{\text{def}}{=} \begin{cases} 0.5q - p & \text{buys} \\ 0.5q & \text{pirates} \\ 0 & \text{does not use,} \end{cases}$$

where q denotes the number of users of this software (which includes the number of buyers and the number of pirates, if piracy takes place). Suppose that the software is costless to produce and costless to protect. Also, assume that TaxMe™ provides support only to those consumers who buy the software.

(i) Suppose that TaxMe™ is *not* copy protected, so piracy is an option for every consumer. Calculate the software seller's profit-maximizing price and the corresponding profit level.

(ii) Suppose that TaxMe™ is copy protected, so piracy is impossible. Calculate the software seller's profit-maximizing price and the corresponding profit level. Does TaxMe™ benefit from protecting its software against piracy?

Set # 5: Telecommunication

(a) Consider an economy with three types of consumers who wish to connect to a certain telecommunication service (e.g., obtaining a phone service). There are 20 type H consumers who place high value for connecting to this service, 10 type M consumers who place a lower value for this connection, and 10 type L consumers who place the lowest value on this service.

Let p denote the connection fee to this service, and q the actual number of consumers connecting to this service. Then, the utility function of each type

$$U_H \stackrel{\text{def}}{=} \begin{cases} 5q - p & \text{connected} \\ 0 & \text{disconnected,} \end{cases} \quad U_M \stackrel{\text{def}}{=} \begin{cases} 2q - p & \text{connected} \\ 0 & \text{disconnected,} \end{cases} \quad U_L \stackrel{\text{def}}{=} \begin{cases} q - p & \text{connected} \\ 0 & \text{disconnected.} \end{cases}$$

Solve the following problems:

(i) Draw the market demand function for connecting to this telecommunication service. Label the axes and prove and explain the graph.

(ii) Suppose now that it costs the telephone company $\mu = 10$ to connect each consumer to this service. Calculate the connection price that maximizes the profit of this monopoly phone company.

(b) Consider an economy with two types of consumers who wish to subscribe to a certain telecommunication service (e.g., obtaining a phone service). There are $\eta_H = 20$ (twenty) type H consumers who place high value for connecting to this service, and $\eta_L = 60$ (sixty) type L consumers who place a low value on this service.

Let p denote the connection fee to this service, and q the actual number of consumers connecting to this service. Then, the utility function of each consumer type

$$U_H \stackrel{\text{def}}{=} \begin{cases} 5q - p & \text{connected} \\ 0 & \text{disconnected} \end{cases} \quad \text{and} \quad U_L \stackrel{\text{def}}{=} \begin{cases} q - p & \text{connected} \\ 0 & \text{disconnected.} \end{cases}$$

(i) Draw the market demand function for connecting to this telecommunication service. Label the axes and prove and explain the graph.

(ii) Suppose now that it costs the telephone company $\mu = 75$ to connect each consumer who subscribes to this service. Calculate the connection price which maximizes the profit of this monopoly phone company.

(c) Consider the demand for a telecommunication service subscription in which consumer types are indexed by x on the interval $[0, 1]$. Suppose there are $\eta = 120$ consumers of each consumer type x. Assume that the utility function of each consumer x, $x \in [0, 1]$ is given by

$$U_x = \begin{cases} (3 - 3x)q^e - p & \text{if she subscribes} \\ 0 & \text{if she does not subscribe,} \end{cases}$$

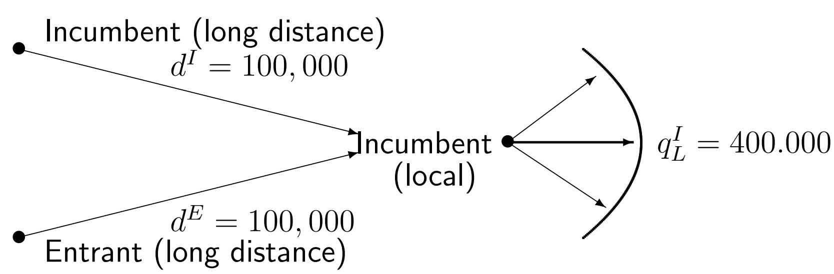
where p is the subscription price, and q^e is the expected number of subscribers.

(i) Formulate and draw the aggregate inverse demand function for this service. Characterize the subscription level under which consumers' willingness to pay reaches the highest level.

(ii) Compute the critical mass, q^{cm}, at the subscription price $p_0 = \$80$.

(iii) Suppose there is only one provider of this telecommunication service. Assume that this monopoly does not bear any cost. Compute the monopoly's profit-maximizing number of subscriptions, the subscription price, p, and the monopoly's profit.

(d) Consider an incumbent monopoly telephone service provider selling long-distance (LD) and local (LC) phone services.



The incumbent provides, both, long-distance and local phone services. Consumers make $d^I = 100,000$ long-distance calls via the incumbent (I), and $d^E = 100,000$ calls via a new entrant (E) who provides only long distance service. Local consumers place $q_L^I = 400,000$ local calls.

The incumbent bears a fixed cost $\phi^I = 1,200,000$ of maintaining the infrastructure for local services. In addition, the incumbent bears a cost $\mu_L^I = 0.5$ for each phone call carried from the *local switch to a local consumer,* that is, the cost of executing a local phone call; and $\mu^I = 3$ which is the cost of a long-distance call carried by the incumbent to the local switchboard. The entrant bears a cost $\mu^E = 2$ of a long-distance call carried by the entrant.

Compute the access charge, a, the entrant must pay the incumbent for each long-distance call originated by the entrant and terminated by the incumbent under two different rules: (1) Fully distributed costs rule, and (2) Efficient component pricing rule (ECPR) assuming that the incumbent charges a price of $p^I = 4$ for a LD phone call. Show the formula of each rule.

(e) Consider a world with two countries labeled N (for North) and S (for South). Country N has η_N consumers who wish to place at most one international phone call to country S. Country S has η_S consumers who wish to place at most one phone call to country N. Assume that $\eta_N > \eta_S$.

Let p_k denote the price of a phone call from country k as charged by the country k's carrier, $k = N, S$. Each potential consumer has a valuation of $\beta > 0$ for placing this phone call, meaning that the utility function of a consumer in country k is given by

$$U_k \stackrel{\text{def}}{=} \begin{cases} \beta - p_k & \text{if makes an international call} \\ 0 & \text{if does not make an international call.} \end{cases}$$

Let a denote the international access charge (settlement rate), which is the payment each carrier makes to the foreign carrier for carrying the phone call to its final destination in the foreign country. Then the profit of each national phone company is composed of profit generated from sales of international phone calls and the collection of access fees from incoming international phone calls. Thus,

$$\pi_N \stackrel{\text{def}}{=} (p_N - a)\eta_N + a\eta_S, \quad \text{and} \quad \pi_S \stackrel{\text{def}}{=} (p_S - a)\eta_S + a\eta_N.$$

Suppose now that the phone industry in country N is fully competitive, hence the price of an international phone call from N to S is $p_N = a$, where a is the negotiated access charge. Also, suppose that the phone industry in country S is a monopoly, hence the price of a phone call from country S to N is $p_S = \beta$. Solve the following problems:

(i) Let a_N be the access charge that maximizes π_N, and let a_S be the access charge that maximizes π_S. Calculate a_N and a_S. Show your calculations.

(ii) Using the bargaining rule $\hat{a} = (a_N + a_S)/2$, calculate the net flow of money transferred from company N to company S. That is, calculate $T_{\vec{NS}}$. Show your calculations!

(iii) Answer questions (i) and (ii) assuming that the phone industry in country N and in country S are both competitive. Show your calculations.

Set # 6: Broadcasting

(a) Consider the broadcasted news scheduling model with three broadcasting stations labeled A, B, and C. There are η viewers whose ideal watching time is 17:00, and η viewers whose ideal watching time is 18:00. Let t_A denote the broadcasting time of station A, t_B the broadcasting time of station B, and t_C the broadcasting time of station C.

Assume that each station can air its news broadcast at one and only one time period. Also assume that each station earns exactly \$1 per viewer (as determined by rating surveys conducted during the broadcasting hours).

Solve the following problems:

(i) List all the Nash equilibria in broadcasting time. (You do not have to provide a formal proof).

(ii) Answer the previous question assuming that there are 3η viewers whose idea watching time is 17:00, and η viewers whose ideal watching time is 18:00.

(b) Consider a scheduling competition between two broadcasting networks labeled A and B which broadcast only evening news. There are 1000 potential viewers. All viewers work full time and therefore can watch the evening news only after they leave work (and not before). More precisely,

100 viewers prefer to watch the news at 18:00 (and not before 18:00 because they must work).

100 viewers prefer to watch the news at 19:00 (and not before 19:00 because they must work).

500 viewers prefer to watch the news at 20:00 (and not before 20:00 because they must work).

300 viewers prefer to watch the news at 21:00 (and not before 21:00 because they must work).

That is, each viewer cannot watch the news before her most-preferred hour, but can watch it after the preferred hour (after finishing work).

Each network must pick the time for its evening news. The profit of network A is $\pi_A(t_A, t_B) = \rho n_A$ where ρ is the revenue per viewer collected from the advertisers, and n_A is the equilibrium number of viewers who watch A. Similarly, $\pi_B(t_A, t_B) = \rho n_B$.

Let t_A denote the broadcasting time of network A, and t_B of network B. Compute the networks' best response functions and conclude which broadcasting times $\langle t_A, t_B \rangle$ constitute a Nash equilibrium. Also, compute the networks' equilibrium profit levels, $\pi_A(t_A, t_B)$ and $\pi_B(t_A, t_B)$.

(c) Consider a program-type competition among 4 independent broadcasting channels: Channels A, B, C, and D. Each channel maximizes the number of viewers times 1¢ (which he receives as a revenue from advertising per viewer). Also assume that that each channel can broadcast only one program type: A talk-show, a news program, or a movie.

There are 3 types of TV viewers: There are 800 viewers who would like to watch only *talk-shows* (T). Similarly, there are 400 viewers who like to watch *news* programs (N) only. Finally, there are 200 viewers who watch *movies* (M) only. Solve the following problems:

(i) Calculate which program will be broadcasted by each channel in a Nash equilibrium. Prove your answer. *Remark*: You do NOT need to demonstrate all NE. You are being asked to prove an existence of one equilibrium).

(ii) Suppose that the utility function of each viewer is given by

$$U = \begin{cases} 5 & \text{if she watches her favorite program} \\ 0 & \text{if she does not watch her favorite program} \end{cases}$$

Define a social welfare function and calculate the amount of social welfare in a Nash equilibrium you found in (5a). Explain whether social welfare is maximized at this equilibrium or whether there is a *market failure*. Prove your answer!

(d) Suppose that there are 4 possible TV program types indexed by $i = 1, 2, 3, 4$. For example, type 1 could be a talk show, type 2 could be the news, type 3 could be a fashion show, and type 4 could be a sports program. Each type of program i is watched by η_i viewers. Suppose that $\eta_1 = 400$, $\eta_2 = 150$, $\eta_3 = 100$, and $\eta_4 = 80$.

Assume that (i) Programs are to be aired in prime time only; hence each broadcasting station can air at most one program type. (ii) If several stations choose to air the same program type, then the program's viewers are evenly split among the stations.

There are three broadcasting stations indexed by $j = A, B, C$. Production is costless. Each station earns a profit of $\$1$ on each viewer, so each station attempts to maximize the number of viewers. We denote by $p_j \in 1, 2, 3, 4$ the action (program type) chosen by station j. Solve the following problems:

(i) Find which type of program will be broadcasted by each station in a Nash equilibrium. You must PROVE your answer using the definition of a Nash equilibrium.

(ii) Suppose that each viewer gains a utility of $U_i = \beta$ if the program of his choice is aired, and $U_i = 0$ if the program of his choice is not aired. Define the social welfare function W by the sum of viewers' utilities. Find the allocation of programs among the three networks that would maximize social welfare. Prove your answer!

(e) Consider a monopoly cable-TV operator providing a service to 3 types of consumers by transmitting 2 channels: CNN and BBC. Assume that the monopoly's production (transmission) is costless and that no royalties are paid to the content providers . The Table below shows the valuation (maximum willingness to pay) of each consumer type for each channel, and the number of consumers of each type. Solve the following problems:

(i) Calculate the profit-maximizing prices assuming that the monopoly must sell each channel separately.

Consumer type	# consumers	CNN	BBC
1	1000	4	1
2	9000	5	5
3	1000	1	4

(ii) Calculate the profit-maximizing price assuming that the monopoly sells all the channels in a single package.

Set # 7: Markets for Information

(a) Consider the library-pricing model analyzed in class. Suppose that there are $\eta = 1200$ potential readers and $\lambda = 50$ libraries (i.e., $1200/50 = 24$ readers per library). The utility function of each potential reader is given by

$$U \stackrel{\text{def}}{=} \begin{cases} 23 - p^b & \text{if she buys and owns the book} \\ 23 - 2p_i^r & \text{if she borrows (rents) from library } i \\ 0 & \text{if she does not read the book.} \end{cases}$$

There is one publisher who can sell either to individual readers, or to libraries but not to both. Each copy of the book costs $\mu = 12$ to produce. Solve the following problems:

(i) Calculate the publisher's profit-maximizing price and her profit level, assuming that the publisher sells directly to individual readers only.

(ii) Calculate the publisher's profit-maximizing library price and her profit level assuming that the publisher sells one copy to each library only.

(b) In the neighborhood where you live, there are $\eta = 12$ residents who are connected to a single Internet service provider (ISP). The capacity of this network is $\bar{Q} = 96$ Mbps (maga-byte per second). Let p denote the price the ISP charges each resident for each 1 Mbps of usage (say, for the amount of downloading). Let q_i denote the demand of resident i (also measured in Mbps). Each resident i ($i = 1, 2, \ldots, 12$) has a utility function given by

$$U_i = \sqrt{q_i} - \left(\frac{1}{2}\right)\left(\frac{Q}{96}\right) - pq_i = \sqrt{q_i} - \frac{1}{2}\,\frac{\sum_{j=1}^{12} q_j}{96} - pq_i,$$

where $Q/96$ reflects how congested the network is. Answer the following questions.

(i) Find how much Internet is demanded by each resident i in a Nash equilibrium assuming that the Internet is provided for free ($p = 0$). Formally, compute q_i. Does not network operate below or above its capacity?

(ii) Compute the socially-optimal Internet usage level of each resident. Does not network operate below or above its capacity when each resident demands the socially-optimal level?

(iii) Compute the price that the ISP should charge (per Mbps) that would induce all resident to demand Internet at the socially-optimal level.

Set # 8: Switching Costs

(a) You are given the following information about a market with two hardware brands labeled A and B:

I. There are 100 A-oriented consumers, and 200 B-oriented consumers.

II. Each consumer type has a utility function given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} q_A - p_A & \text{buy } A \\ q_B - p_B - \delta & \text{buy } B, \end{cases} \quad U_B \stackrel{\text{def}}{=} \begin{cases} q_A - p_A - \delta & \text{buy } A \\ q_B - p_B & \text{buy } B, \end{cases}$$

where δ is the differentiation (switching cost) parameter.

III. Production is costless and in an undercut-proof equilibrium, brands' prices are

$$p_A = \frac{1485}{7} \quad \text{and} \quad p_B = \frac{1188}{7}.$$

Calculate the differentiation (switching-cost) parameter δ.

(b) Consider a city with only two local banks labeled bank A and bank B. Each person is allowed to maintain only one bank account (either with bank A, or B, but not both). Let δ_A denote the cost of switching a bank account from bank A to bank B. That is, the total cost (including inconvenience) of closing an account with bank A and opening a fully-operative account with bank B. Similarly, let δ_B denote the cost of switching a bank account from bank B to bank A.

You are now given the following information:

I. Bank A maintains 100 (one hundred) accounts, whereas bank B maintains 200 (two hundred) accounts.

II. Bank A levies a fee of $f_A = 30$ per account and bank B levies a fee of $f_B = 30$ per account.

III. The utility functions of a bank A and a bank B account holder, respectively, are given by

$$U_A \stackrel{\text{def}}{=} \begin{cases} -f_A & \text{staying with bank } A \\ -f_B - \delta_A & \text{switching to bank } B, \end{cases} \quad U_B \stackrel{\text{def}}{=} \begin{cases} -f_A - \delta_B & \text{switching to bank } A \\ -f_B & \text{staying with bank } B. \end{cases}$$

Suppose that banks do not bear any costs and that their fees are set in an undercut-proof equilibrium. Using the above data, calculate the switching-cost parameters δ_A and δ_B. Show your calculations!

(c) In Ben Barbor there are three banks. Each resident (depositor) has one account in one (and only one) of the banks. Bank A has $n_A = 600$ depositors. Bank B has $n_B = 400$ depositors. Bank C has $n_C = 200$ depositors. Assume that the banks bear no cost of serving customers.

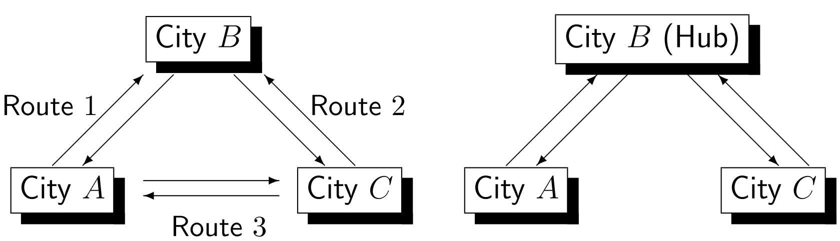
It turns out that in an UPE all banks charge each depositor the same annual fee of $\$60$. That is, $f_A = f_B = f_C = \$60$. The utility function of a depositor who has an account with bank i ($i = A, B, C$) is given by

$$U_i = \begin{cases} -f_i & \text{staying with bank } i \\ -f_j - \delta_i & \text{switching from bank } i \text{ to bank } j. \end{cases}$$

Using the above data, compute the switching cost parameters δ_A, δ_B, and δ_C. Show your derivations.

Set # 9: The Airline Industry

(a) A single airline company serves 3 cities as illustrated in the following figure.



On each route i, $i = 1, 2, 3$, there are η_i passengers. The cost of operating a flight on each route i is given by the function $c(q_i) = 1000 + \sqrt{q_i}$, where q_i is the actual number of passengers flying on route i. That is, each route requires a fixed cost of 1000 independent of the number of passengers plus variable cost which depends on the number of passengers q_i.

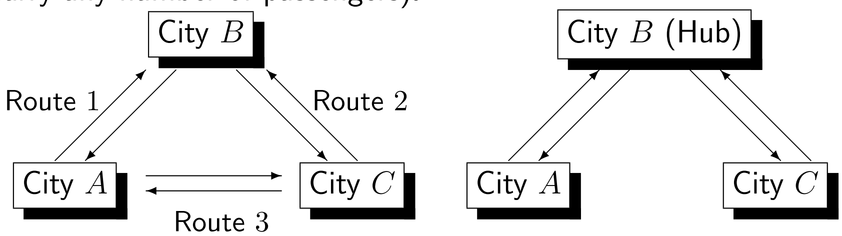
The airline considers two alternative networks of operations (displayed in the above figure): A fully-connected (FC), and a Hub-and-Spoke (with a hub in city B).

Solve the following problems:

(i) $\eta_1 = \eta_2 = \eta_3 = \eta > 0$. Calculate which network of operation (FC or HS) minimized the airline's cost. Show your calculation!

(ii) Suppose that the airline has already decided to operate a Hub-and-Spoke network. Also suppose that $\eta_1 = 100$ and $\eta_2 = \eta_3 = 800$. Does the airline minimize cost by locating the hub at city B? Prove your answer!

(b) A single airline company serves 3 cities as illustrated in the following figure. The cost of operating a flight on each route i is $\mu = 200$. Assume that aircrafts have an unlimited capacity (can carry any number of passengers).



On <u>each</u> route i, $i = 1, 2, 3$, there are $\eta_i^H = 50$ passengers who have high value of time, and $\eta_i^L = 10$ passengers who have low value of time. The utility functions of type H and L passengers on route i are

$$U_i^H \stackrel{\text{def}}{=} \begin{cases} 12 - p_i & \text{flies directly to destination} \\ 8 - p_i & \text{flies to destination via a hub} \\ 0 & \text{does not fly,} \end{cases} \quad \text{and} \quad U_i^L \stackrel{\text{def}}{=} \begin{cases} 12 - p_i & \text{flies directly or indirectly} \\ 0 & \text{does not fly.} \end{cases}$$

(i) Compute the profit-maximizing airfare on each route $i = 1, 2, 3$, assuming that the airline operates a Hub-and-Spokes (HS) network. Also compute total profit of this operator.

(ii) Compute the profit-maximizing airfare on each route $i = 1, 2, 3$ assuming that the airline operates a Fully-connected (FC) network. Also compute total profit of this operator, and determine which network of operation is more profitable for this monopoly airline.

(c) Consider two airline companies: airline α and airline β, who are the only airlines providing a service connecting city A with city B. Suppose that the frequency of flights provided by airline α and airline β are $f_\alpha = 6$ and $f_\beta = 3$, respectively. That is, airline α provides 6 flights per day, whereas airline β provides only 3 flights per day. There are η consumers who are oriented towards airline α, and η who are oriented towards airline β. Suppose now that passengers' utility functions are given by:

$$U_\alpha \stackrel{\text{def}}{=} \begin{cases} f_\alpha - p_\alpha & \text{flies } \alpha \\ f_\beta - 4 - p_\beta & \text{flies } \beta, \end{cases} \quad \text{and} \quad U_\beta \stackrel{\text{def}}{=} \begin{cases} f_\alpha - 4 - p_\alpha & \text{flies } \alpha \\ f_\beta - p_\beta & \text{flies } \beta. \end{cases}$$

Assume that the airline firms do not bear any type of cost. Answer the following questions.

(i) Calculate the UPE airfare charged by each airline and the associated profit levels assuming that there are no agreements between the two airline firms.

(ii) Calculate the UPE airfares and the associated profit levels assuming that the two airline firms are engaged in a code-sharing agreement. Assume that airline α continues to maintain $f_\alpha = 6$ flights per day and airline β continues to maintain $f_\beta = 3$ flights per day even after the agreement is signed.

Set # 10: Languages as Networks

(a) In an Island named *Bilingwa* off the coast of Mexico there are 100 inhabitants. $\eta_E = 60$ are native English speakers, whereas $\eta_S = 40$ are native Spanish speakers. Let n_{ES} denote the number of native English speakers who learn to speak Spanish. Similarly, let n_{SE} denote the number of native Spanish speakers who learn English. The utility of each resident increases with the number of residents to whom he is able to communicate with. We define the utility function of each native English and each native Spanish speakers, respectively, by

$$U_E = \begin{cases} \dfrac{60 + n_{SE}}{10} & \text{does not learn Spanish} \\ \dfrac{60 + 40}{10} - 3 & \text{learns Spanish,} \end{cases} \qquad U_S = \begin{cases} \dfrac{40 + n_{ES}}{10} & \text{does not learn English} \\ \dfrac{40 + 60}{10} - 7 & \text{learns English.} \end{cases}$$

These utility functions reveal that it is "easier" (less costly) for a native English speaker to learn Spanish, than for a native Spanish speaker to learn English (cost of $\phi_S = 3$ compared with $\phi_E = 7$).

Solve the following problems:

(i) Find the number of native English speakers who learn Spanish, n_{ES}, and the number of native Spanish speakers who learn English, n_{SE} in a language-acquisition equilibrium. Is the equilibrium you found unique? Prove your results!

(ii) Find the socially-optimal levels of n_{ES} and n_{SE}. Prove your answer!

(b) In an Island named *Bilingwa* off the coast of Mexico there are 100 inhabitants. $\eta_E = 60$ are native English speakers, whereas $\eta_S = 40$ are native Spanish speakers. Let n_{ES} denote the number of native English speakers who learn to speak Spanish. Similarly, let n_{SE} denote the number of native Spanish speakers who learn English. The utility of each resident increases with the number of residents to whom he is able to communicate with. We define the utility function of each native English and each native Spanish speakers, respectively, by

$$U_E = \begin{cases} \dfrac{60 + n_{SE}}{10} & \text{does not learn Spanish} \\ \dfrac{60 + 40}{10} - 5 & \text{learns Spanish} \end{cases} \qquad U_S = \begin{cases} \dfrac{40 + n_{ES}}{10} & \text{does not learn English} \\ \dfrac{40 + 60}{10} - 7 & \text{learns English} \end{cases}$$

These utility functions reveal that it is "easier" (less costly) for a native English speaker to learn Spanish, than for a native Spanish speaker to learn English (cost of $\phi_S = 5$ compared with $\phi_E = 7$).

(i) Find the number of native English speakers who learn Spanish, n_{ES}, and the number of native Spanish speakers who learn English, n_{SE} in a language-acquisition equilibrium. Prove your results!

(ii) Find the socially-optimal levels of n_{ES} and n_{SE}. Prove your answer!

(c) In a small island in South-East Asia there are 60 native Bengali speakers and 40 Hindi speakers. The language school on this island teaches 3 separate classes: Bengali (language B), Hindi (language H), and English (language E). The tuition is $30 for each class.

Denote by n_{BH} the number of Bengali speakers who learn Hindi; n_{BE} the number of Bengali speakers who learn English; n_{HB} the number of Hindi speakers who learn Bengali; n_{HE} the number of Hindi speakers who learn English. The utility functions of a Bengali and an Hindi native speakers are

$$U_B = \begin{cases} 60 + n_{HB} & \text{does not study} \\ 100 - 30 & \text{studies Hindi} \\ 60 + n_{HB} + n_{HE} - 30 & \text{studies English} \end{cases} \quad U_H = \begin{cases} 40 + n_{BH} & \text{does not study} \\ 100 - 30 & \text{studies Bengali} \\ 40 + n_{BH} + n_{BE} - 30 & \text{studies English} \end{cases}$$

(i) State whether the following statement is True or False (prove): There exists an equilibrium in which all the island's residents learn English.

(ii) Compute aggregate consumer welfare levels corresponding to the following three separate situations: I. All residents learn English. II. All Bengali speakers learn Hindi. III. All Hindi speakers learn Bengali. Conclude which situation yields the highest aggregate consumer welfare.

THE END

