(1a) [10 points] Firm A solves

$$\max_{q_A} \pi_A = 80q_A - \frac{3}{2}(q_A)^2 - q_B q_A \quad \Longrightarrow \quad 0 = \frac{\partial \pi_A}{\partial q_A} = 80 - 3q_A - q_B.$$

Firm B solves

$$\max_{q_B} \pi_B = 80q_B - \frac{3}{2}(q_B)^2 - q_A q_B \quad \Longrightarrow \quad 0 = \frac{\partial \pi_B}{\partial q_B} = 80 - 3q_B - q_A.$$

Second-order conditions are clearly satisfied. Solving two equations with two variables yields $q_A^c = q_B^c =$ 20 units. Hence, $p_A^c = p_B^c = \$30$, and $\pi_A^c = \pi_B^c = 30 \cdot 20 = \$600$.

(1b) [5 points]

$$p_B = 80 - \frac{3}{2}q_B - q_A \quad \Longrightarrow \quad q_A = 80 - \frac{3}{2}q_B - p_B \quad \Longrightarrow \quad p_A = 80 - \frac{3}{2}\left(80 - \frac{3}{2}q_B - p_B\right) - q_B$$

Hence,

$$p_A = -40 + \frac{5}{4}q_B + \frac{3}{2}p_B \quad \Longrightarrow \quad \frac{5}{4}q_B = p_A + 40 - \frac{3}{2}p_B \quad \Longrightarrow \quad q_B = 32 - \frac{6}{5}p_B + \frac{4}{5}p_A.,$$

which is the direct demand function for brand B. Next, substituting the direct demand function for q_B into the second equation above.

$$q_A = 80 - \frac{3}{2}\left(32 + \frac{4}{5}p_A - \frac{6}{5}p_B\right) - p_B = 32 - \frac{6}{5}p_A + \frac{4}{5}p_B.$$

(1c) [5 points] Firm A chooses p_A to solve

$$\max_{p_A} \pi_A = 32p_A - \frac{6}{5}(p_A)^2 + \frac{4}{5}p_B p_A \quad \Longrightarrow \quad = \frac{\partial \pi_A}{\partial p_A} = 32 - \frac{12}{5}p_A + \frac{4}{5}p_B.$$

Firm B chooses p_B to solve

$$\max_{p_B} \pi_B = 32p_B - \frac{6}{5}(p_B)^2 + \frac{4}{5}p_B p_A \quad \Longrightarrow \quad = \frac{\partial \pi_B}{\partial p_B} = 32 - \frac{12}{5}p_B + \frac{4}{5}p_A.$$

Solving 2 equations with 2 variables yields $p_A^b = p_B^b = \$20$. Substituting into the direct demand functions yields $q_A^b = q_B^b = 24$ units. Hence, $\pi_A^b = \pi_B^b = 20 \cdot 24 = \$480$.

A comparison of the quantity game with the price game implies

$$p_i^b = \$20 < \$30 = p_i^c, \quad q_i^b = 24 > 20 = q_i^c, \quad \text{and} \quad \pi_i^b = \$480 < \$600 = \pi_i^c, \quad \text{for firm } i = A, B.$$

Thus, competition is more intense when firm play a price game compared with a quantity game.

(2a) [10 points] All the firms have identical R&D technologies. Hence, it is sufficient to compute the profit of one representative firm, say firm 1. The profit of firm 1 when all 3 firms engage in R&D is

$$\pi_1(3) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\frac{240}{3} + 2\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\frac{240}{2} + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)240 - 120 = \frac{10}{3} > 0.$$

Hence, in equilibrium 3 firms will enter the R&D patent race.

(2b) [10 points] Expected profit when the investor operates all 3 labs is:

$$\pi^s(3) = 640\left[1-\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\right] - 3\cdot 120 = \$10.$$

Expected profit when the investor operates 2 labs is:

$$\pi^s(2) = 640\left[1-\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\right] - 2\cdot 120 = \$40.$$

Expected profit when the investor operates 1 lab is:

$$\pi^s(1) = 640\left(\frac{1}{4}\right) - 120 = \$40.$$

Hence, under single ownership, the owner will choose to operate either 2 labs or 1 lab only.

(3a) [5 points] A monopoly, extracting entire consumer surplus, will set the price on the basis of \$20 per month of service. Hence, $p_L = 30\cdot 60 = \$1800$ and $p_S = 30\cdot 40 = \$1200$. The profits per month of use are:

$$\frac{\pi_L}{60} = 30 - \frac{c_L}{60} = 30 - \frac{120}{60} = \$28 \quad \text{and} \quad \frac{\pi_S}{40} = 30 - \frac{c_S}{40} = 30 - \frac{80}{40} = \$28.$$

Hence, a monopoly seller would be indifferent between selling long and short lasting batteries (or both types).

(3b) [5 points] In a competitive industry, prices drop to marginal costs. Hence, $p_L = \$120$ and $p_S = \$80$. Consumers' utility (per month of service) from each battery type are given by

$$\frac{U_L}{60} = 30 - \frac{p_L}{60} = 30 - \frac{120}{60} = 28 \quad \text{and} \quad \frac{U_S}{40} = 30 - \frac{p_S}{40} = 30 - \frac{80}{40} = 28.$$

Hence, both types of batteries will be demanded when produced by an competitive industry.

(4a) [4 points]

$$p^{NW} = \rho V = \frac{3}{4}120 = \$90. \quad \text{hence} \quad \pi^{NW} = 90 - 60 = \$30.$$

(4b) [6 points] Since consumers are fully protected against defects, they are willing to pay $p^W = \$120$, which is the price charged by a monopoly seller. The monopoly's expected cost is $C + (1-\rho)R$. Hence, the monopoly profit is

$$\pi^W = p^W - C - (1-\rho)R = 120 - 60 - \frac{1}{4}40 = \$50 > \$30 = \pi^{NW}.$$

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(5) [20 points] Try first assuming that Day is the "high season." Therefore,

$$MR_D = 12 - q_D = r = 4 \implies q_D = K = 8 \implies p_D = 12 - \frac{8}{2} = 8.$$

For the Night "season"

$$MR_N = 24 - 4q_N = c = 0 \implies q_N = 6 \implies p_N = 24 - 2 \cdot 6 = 12.$$

It is important to confirm that Day is indeed the high season by verifying that $q_N = 6 < 8 = K$, so the night demand can be accommodated with the capacity K.

(6a) [10 points] With no tying, pricing R at a high rate so that only type 1 guests book a room, $p_R = \$100$ yields a profit of $\pi_R = (100 - 40)200 = \$12,000$. Reducing the price so that both types book a room, $p_R = \$60$ yields a profit of $\pi_R = (60 - 40)1000 = \$20,000$. Therefore, $p_R = \$60$ is the profit-maximizing rate.

Setting a high breakfast price so that only type 2 consumers buy breakfast, $p_B = \$10$, yields a profit of $\pi_B = (10 - 2)800 = \$6400$. Reducing the price so that both types buy, $p_B = \$5$, yields a profit of $\pi_B = (5 - 2)1000 = \$3000$. Therefore, $p_B = \$10$ is the profit-maximizing breakfast price.

The gym should be priced at $p_G = \$10$, yielding a profit of $\pi_G = \$10,000$. Altogether, total profit with no tying is $\pi^{NT} = 20,000 + 6400 + 10,000 = \$36,400$.

(6b) [10 points] Setting a high price for the package, $p_{RBG} = \$115$, attracts only 200 customers, hence yields a profit of $\pi^{PT} = (115 - 42)200 = \$14,600$. Setting a low price, $p_{RBG} = \$80$, attracts all 1000 customers, hence yields a profit of $\pi^{PT} = (80 - 42)1000 = \$38,000 > \pi^{NT}$. Therefore, pure tying is more profitable than no tying.

THE END

