(1) Two firms have technologies for producing identical paper clips. Assume that all paper clips are sold in boxes containing 100 paper clips. Firm A can produce each box at unit cost of $c_A = \$6$ whereas firm B (less efficient) at a unit cost of $c_B = \$8$.

(1a) [10 pts.] Suppose that the aggregate market demand for boxes of paper clips is $p = 12 - Q/2$, where p is the price per box and Q is the number of boxes sold. Solve for the Nash-Bertrand equilibrium prices p^b_A and p^b_B, and the equilibrium profits π^b_A and π^b_B. Explain your reasoning!

Answer: The first case to be checked is where the efficient firm A undercuts B by setting $p_A = c_B - \epsilon = 8 - \epsilon$, where ϵ is a small number. Firm B sets $p_B = c_B = \$8$. In this case, all consumers buy brand A only, hence, solving $8 = 12 - q_A/2$ yields $q_A = 8$ and $q_B = 0$. The profits are then $\pi_A = (8-6)8 = \$16$ and $\pi_B = 0$.

The second case to be checked is where A sets a monopoly price. Solving $MR = 12 - q_A = c_A = 6$ yields $q_A = 6$. Hence, $p = 12 - 6/2 = \$8 > \$8 = p_B$. Therefore, in this case, firm A cannot charge its monopoly price.

Altogether, a Nash-Bertrand equilibrium is $p^b_A = \$8 - \epsilon$ and $p^b_B = \$8$. The equilibrium profits are therefore $\pi^b_A = (8-6)8 = \$16$ and $\pi^b_B = 0$.

(1b) [10 pts.] Answer the previous question assuming that firm A has developed a cheaper production technology so its unit cost is now given by $c_A = \$2$.

Answer: The first case to be checked is where the efficient firm A undercuts B by setting $p_A = c_B - \epsilon = 8 - \epsilon$, where ϵ is a small number. Firm B sets $p_B = c_B = \$8$. In this case, all consumers buy brand A only, hence, solving $8 = 12 - q_A/2$ yields $q_A = 8$ and $q_B = 0$. The profits are then $\pi_A = (8-2)8 = \$48$ and $\pi_B = 0$.

The second case to be checked is where A sets a monopoly price. Solving $MR = 12 - q_A = c_A = 2$ yields $q_A = 10$. Hence, $p = 12 - 10/2 = \$7 < \$8 = p_B$. Therefore, $\pi_A = (7-2)10 = \$50 > \$48$.

Altogether, a Nash-Bertrand equilibrium is $p^b_A = \$7$ and $p^b_B = \$8$. The equilibrium profits are therefore $\pi^b_A = (7-2)10 = \$50$ and $\pi^b_B = 0$.

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(2) Consider an infinitely-repeated price competition game between GM and FORD. Each firm can set a high price or a low price in each period $t = 0, 1, 2, \ldots$. The profit of each outcome are given in the following matrix:

		FORD			
		LOW PRICE (p^L)		HIGH PRICE (p^H)	
GM	LOW (p^L)	4	3	5	1
	HIGH (p^H)	1	6	5	4

Suppose that each firm adopts a trigger-price strategy under which the firms may be able to implicitly collude on setting the high price. Let ρ $(0 < \rho < 1)$ denote the time discount factor.

(2a) [10 pts.] Compute the minimum threshold value of ρ which would ensure that GM sets p^H in every period t. Show and explain your derivations.

Answer: GM's discounted sum of profits when it does not deviate from the collusive high price, and when it deviates from the collusive price are given by

$$\pi_G = \frac{5}{1-\rho} \quad \text{and} \quad \pi'_G = 5 + \rho\,\frac{4}{1-\rho}.$$

Hence, $\pi_G \geq \pi'_G$ for every ρ satisfying $0 < \rho < 1$. Intuitively, it follows directly from the profit levels in the above table that GM cannot benefit even from one-period deviation since $\pi_G(p^L, p^H) = 5 = \pi_G(p^H, p^H)$.

(2b) [10 pts.] Compute the minimum threshold value of ρ which would ensure that FORD sets p^H in every period t. Show and explain your derivations.

Answer: Ford's discounted sum of profits when it does not deviate from the collusive high price, and when it deviates from the collusive price are given by

$$\pi_F = \frac{4}{1-\rho} \quad \text{and} \quad \pi'_F = 6 + \rho\,\frac{3}{1-\rho}.$$

Hence, $\pi_F \geq \pi'_F$ if $\rho > 2/3$.

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(3) Aike (Brand A) and Beebok (Brand B) are leading brand names of fitness shoes. The direct demand functions facing each producer are given by

$$q_A(p_A, p_B) = 180 - 2p_A + p_B \quad \text{and} \quad q_B(p_A, p_B) = 120 - 2p_B + p_A.$$

Assume zero production cost ($c_A = c_B = 0$).

(3a) [10 pts.] Derive the price best-response function of firm A as a function of the price set by firm B, $p_A = BR_A(p_B)$. Show your derivations, and draw the graph associated with this function.

Answer: For a given p_B, firm A chooses p_A to solve

$$\max_{p_A} \pi_A = p_A q_A = (180 - 2p_A + p_B) \Longrightarrow 0 = \frac{\mathrm{d}\pi_A}{\mathrm{d}p_A} = 180 - 4p_A + p_B \Longrightarrow p_A = BR_A(p_B) = 45 + \frac{1}{4}p_B.$$

(3b) [10 pts.] Derive the price best-response function of firm B as a function of the price set by firm A, $p_B = BR_B(p_A)$. Show your derivations, and draw the graph associated with this function.

Answer: For a given p_A, firm B chooses p_B to solve

$$\max_{p_B} \pi_B = p_B q_B = (120 - 2p_B + p_A) \Longrightarrow 0 = \frac{\mathrm{d}\pi_B}{\mathrm{d}p_B} = 120 - 4p_B + p_A \Longrightarrow p_B = BR_B(p_A) = 30 + \frac{1}{4}p_A.$$

(3c) [10 pts.] Solve for the Nash-Bertrand equilibrium prices, $\langle p_A^b, p_B^b \rangle$. Then, compute the equilibrium output levels $\langle q_A^b, q_B^b \rangle$, the equilibrium profits $\langle \pi_A^b, \pi_B^b \rangle$, and aggregate industry profit $\Pi^b = \pi_A^b + \pi_B^b$.

Answer: Solving the above two best-response functions yields $p_A^b = \$56$ and $p_B^b = \$44$. Substituting prices into the direct demand functions yields

$$q_A^b = 180 - 2 \cdot 56 = 44 = 112 \quad \text{and} \quad q_B^b = 120 - 2 \cdot 44 + 56 = 88.$$

Hence, $\pi_A^b = 56 \cdot 112 = \$6272$ and $\pi_B^b = 44 \cdot 88 = \$3872$. Finally, aggregate industry profit is: $\Pi^b = \pi_A^b + \pi_B^b = \$10,144$.

(3d) [10 pts.] Suppose now that the two producers hold secret meetings in which they discuss fixing the price of shoes to a uniform (brand-independent) level of $p = p_A = p_B$. Compute the price p which maximizes joint industry profit, $\pi_A + \pi_B$. Then, compute aggregate industry profit and compare it to the aggregate industry profit made under Bertrand competition which you computed in part (3c).

Answer: Setting $p = p_A = p_B$, this cartel's joint profit is

$$\pi_A + \pi_B = (180 - 2p + p)p + (120 - 2p + p) = 300p - 2p^2.$$

Maximizing $\pi_A + \pi_B$ with respect to p yields

$$0 = \frac{\mathrm{d}\pi_A + \pi_B}{\mathrm{d}p} = 300 - 4p \Longrightarrow p = \$75 \Longrightarrow \pi_A + \pi_B = \$11,250 > \$10,144$$

which is the aggregate industry profit earned under Bertrand competition.

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(4) Ann Arbor and Ypsilanti are very similar cities, because each city has exactly one McDonald's. Ann Arbor has $N_A = 200$ residents and Ypsilanti has $N_Y = 200$ residents. Each resident demands one hamburger. A resident of Ann Arbor who wishes to buy a hamburger in Ypsilanti must bear a transportation cost of $T_A = \$3$. Similarly, a resident of Ypsilanti who wishes to buy a hamburger in Ann Arbor must bear a transportation cost of $T_Y = \$3$.

(4a) [10 pts.] Solve for the undercut-proof equilibrium prices p_A^U and p_Y^U and profit levels π_A^U and π_Y^U assuming that McDonald's has the technology for producing hamburgers at no cost. Show your derivation.

Answer: In an UPE, store A sets the highest p_A subject to

$$\pi_Y = 200 p_Y \geq (200 + 200)(p_A - 3).$$

Similarly, store Y sets the highest p_Y subject to

$$\pi_A = 200 p_A \geq (200 + 200)(p_Y - 3).$$

Solving two equations with two variables for the case of equality, yields $p_A^U = \$6$ and $p_Y^U = \$6$. Hence, $\pi_A^U = 200 \cdot 6 = \$1200$ and $\pi_Y^U = 200 \cdot 6 = \$1200$.

(4b) [10 pts.] Answer the previous question assuming now that McDonald's in Ann Arbor bears a cost of $\$1$ of producing each hamburger, whereas McDonald's in Ypsilanti bears a cost of $\$4$ of producing each hamburger. Show your derivation.

Answer: In an UPE, store A sets the highest p_A subject to

$$\pi_Y = 200(p_Y - 4) \geq (200 + 200)(p_A - 4 - 3).$$

Similarly, store Y sets the highest p_Y subject to

$$\pi_A = 200(p_A - 1) \geq (200 + 200)(p_Y - 1 - 3).$$

Solving two equations with two variables for the case of equality, yields $p_A^U = \$9$ and $p_Y^U = \$8$. Hence, $\pi_A^U = 200(9-1) = \$1600$ and $\pi_Y^U = 200(8-4) = \$800$.

THE END