(1a) [10 points] The gross consumer surplus of consumer type 1 and type 2 when they consumer q units are

$$gcs_1(q) = \frac{8+p}{2}\,q = \frac{8+8-2q}{2}\,q = q(8-q), \quad \text{for } q \le 4; \quad gcs_1(q) = 16 \quad \text{for } q \ge 4 \quad \text{and}$$

$$gcs_2(q) = \frac{4+p}{2}\,q = \frac{8+4-0.5q}{2}\,q = \frac{q(16-q)}{4} \quad \text{for } q \le 8.$$

Using the above formulas, If a bundle contains exactly $q^b = 3$ units, $gcs_1(3) = 15$ and $gcs_2(2) = 39/4 < 15$. Setting a low price $p^b = 39/4$ implies that both consumers buy this bundle. Hence, profit is $y_{1,2} = 2(39/4 - 2\cdot 3) = 15/2$. Setting a high price $p^b = 15$ implies that only type 1 buys this bundle. Hence, profit is $y_1 = 15 - 2\cdot 3 = 9 > 15/2$.

If a bundle contains $q^b = 4$ units, $gcs_1(4) = 16$ and $gcs_2(4) = 12$. Setting a low price $p^b = 12$ implies that both consumers buy this bundle. Hence, profit is $y_{1,2} = 2(12 - 2\cdot 4) = 8$. Setting a high price $p^b = 16$ implies that only type 1 buys this bundle. Hence, profit is $y_1 = 16 - 2\cdot 4 = 8$.

The above computations reveal that the profit-maximizing contains $q^b = 3$ units, sold for a price $p^b = 15¢$, thus generating a profit of $y_1 = 9¢$.

(1b) [5 points] Both consumers will buy the 5-stick pack because

$$gcs_1(2) - 10 = 12 - 10 = 2 < 3 = 16 - 13 = gcs_1(5) - 13$$
$$gcs_2(2) - 10 = 7 - 10 = -3 < 0.75 = \frac{55}{4} - 13 = gcs_1(5) - 13$$

The resulting profit is $y = 2(13 - 2\cdot 5) = 6¢ < 9¢$. Hence, selling these two packs won't enhance the producer's profit because both consumers end up buying the same packages (no market segmentation).

Remark: $gcs_1(5) = gcs_1(4) = 16$.

(1c) [5 points] For $p \le 4¢$, the aggregate demand facing the producer is

$$Q = q_1 + q_2 = \frac{8-p}{2} + 2(4-q) = \frac{24-5p}{2} \quad \text{or} \quad p = \frac{2(12-Q)}{5}.$$

For $p \le 4$, the monopoly solves

$$MR = \frac{2(12-2Q)}{5} = 2 = \mu \quad \text{yielding } Q = \frac{7}{2}, \quad p = \frac{17}{5} < 4.$$

Hence,

$$y_{1,2} = \left(\frac{17}{5} - 2\right)\frac{7}{2} = \frac{49}{10} = 4.9¢.$$

Selling at a price $p > 4¢$ would exclude consumer 2 from the market. In this case, the monopoly solves

$$MR_1 = 8 - 4q_1 = 2¢ = \mu, \quad \text{yielding } q_1 = \frac{3}{2} \quad \text{and} \quad p_1 = 5¢ > 4¢.$$

Under this price, the monopoly earns

$$y_1 = (5-2)\frac{3}{2} = \frac{9}{2} = 4.5¢ < 4.9¢.$$

Hence, the profit maximizing price is $p = 17/5 = 3.4¢$.

---

(2) [20 points] See textbook Exercise 3.9 on p.112. Notice an error in the solution for part (b) on p.386. The correct answer is $q_2 = 230$ (instead of $q_2 = 240$). For this reason, I gave full credit to both solutions.

(3a) [5 points] To break even, we solve

$$y_1 = (120 - 40)70 = (110 - 40)(q_1 + \Delta q) \quad \text{yielding} \quad \Delta q = 10.$$

Alternatively, you can use the break even formula

$$\Delta q = \frac{-q_1 \Delta p}{p_1 + \Delta p - \mu} = \frac{-70(-10)}{120 - 10 - 40} = 10.$$

(3b) [5 points] The firm earns non-negative profits if

$$y = (p - \mu)q - \phi = (110 - 40)q - 3500 \geq 0 \quad \text{iff} \quad q \geq 50 \text{ units.}$$

---

(4) [10 points] *Remark*: The figures here are the same as on the third column of Table 5.3 on p.162, with the demand functions taken from equation (5.12) on p.160.

At $p = \$2$ per ride, a type 1 visitor buys $q_1 = (8-2)/2 = 3$ rides. A type 2 visitor buys $q_2 = 2(4-2) = 4$ rides. Next,

$$gcs_1(3) = \frac{8+2}{2}3 = 15 \quad \text{and} \quad gcs_2(4) = \frac{4+2}{2}4 = 12.$$

Therefore, the maximal admission (fixed) fee that can be charged from each consumer type is

$$f_1 = gcs_1(3) - 2 \cdot 3 = 15 - 6 = \$9 \quad \text{and} \quad f_2 = gcs_2(4) - 2 \cdot 4 = 12 - 8 = \$4.$$

Setting the "low" fixed fee, $f = \$4$ yields a profit

$$y_{1,2} = 2[4 + 3(2-2)] + 5[4 + 4(2-2)] = \$28.$$

Setting the "high" fixed fee, $f = \$9$ yields a profit

$$y_1 = 2[9 + 3(2-2)] = \$18 < \$28.$$

Hence, the profit-maximizing admission fee is $f = \$4$.

---

(5) [20 points] See Exercise 2 on page 223 (solution on pp.396–397).

---

(6) [20 points] See Exercise 1 on page 355 (solution on pp.414–415).

THE END

