# Lecture 13
# Game Theory I: Introduction

15.011/011 Economic Analysis for Business Decisions

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# What is a game?

Before giving a "formal" definition, let's look at an example of a normal-form game (a single-stage game in a matrix format)

| | | Firm 2 | |
|---|---|---|---|
| | $a_1$ / $a_2$ | Low Price (L) | High Price (H) |
| Firm 1 | Low Price (L) | 100 100 | 300 0 |
| | High Price (H) | 0 300 | 200 200 |

<u>Definition</u>: A game is:

1. A list of players' names: Firm 1 and Firm 2
2. A strategy set of each player (list of actions): $S_1$ = {Low, High} and $S_2$ = {Low, High}
   Need not always be the same for each player
3. Payoff (Profit) functions (for each of the 4 possible outcomes)

# What is a game?

Firm 2

| $a_1$ / $a_2$ | Low Price (L) | High Price (H) |
|---|---|---|
| Firm 1 Low Price (L) | 100 100 | 300 0 |
| High Price (H) | 0 300 | 200 200 |

- This game has 4 possible outcomes of this game: (Low, Low), (Low, High), (High, Low), and (High, High)
- The Economist's job is to predict what the market outcome would be realized
- For that we need an "equilibrium concept"

But, there are several equilibrium concepts, that may yield different predictions! We'll discuss a few

# A powerful tool: Best-response functions

Firm 2

| $a_1$ / $a_2$ | Low Price (L) | | High Price (H) | |
|---|---|---|---|---|
| Low Price (L) | 100 | 100 | 300 | 0 |
| High Price (H) | 0 | 300 | 200 | 200 |

Firm 1

$$BR_1(a_2) = \begin{cases} L & \text{if } a_2 = L \\ L & \text{if } a_2 = H \end{cases} \qquad BR_2(a_1) = \begin{cases} L & \text{if } a_1 = L \\ L & \text{if } a_1 = H \end{cases}$$

That is, Firm 1 will choose *L* if Firm 2 chooses to "play" action *L*. Also, Firm 1 will choose *L* if Firm 2 chooses action *H*

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Remark: For our purposes, in single-stage games, a "strategy" and "action" would mean the same thing

# Dominant strategy (action) for a player & equilibrium in dominant strategies

$$BR_1(a_2) = \begin{cases} L & \text{if } a_2 = L \\ L & \text{if } a_2 = H \end{cases} \qquad BR_2(a_1) = \begin{cases} L & \text{if } a_1 = L \\ L & \text{if } a_1 = H \end{cases}$$

If Firm 1 chooses one action regardless of the action chosen by the rival firm, then Firm 1 has a dominant strategy (action)

In this game: *L* is a dominant strategy of Firm 1.
Also, *L* happens to be a dominant strategy of Firm 2.

If each player has a dominant strategy, then an equilibrium in dominant strategies exists.

Therefore,
(*L*, *L*) is an equilibrium in dominant strategies (dominant actions)

Remark: In equilibrium, each firm earns $100. However, if they were able to collude, they could earn $200 each! (Prisoner's' Dilemma)

# Non-existence of an equilibrium in dominant strategies

Firm 2

| | $a_1$ / $a_2$ | Standard α | | Standard β | |
|---|---|---|---|---|---|
| Firm 1 | Standard α | 200 | 100 | 0 | 0 |
| | Standard β | 0 | 300 | 300 | 200 |

$$BR_1(a_2) = \begin{cases} \alpha & \text{if } a_2 = \alpha \\ \beta & \text{if } a_2 = \beta \end{cases} \qquad BR_2(a_1) = \begin{cases} \alpha & \text{if } a_1 = \alpha \\ \alpha & \text{if } a_1 = \beta \end{cases}$$

That is, Firm 1 does not have a dominant strategy!
Hence, an equilibrium in dominant strategies does not exist!

Remark: We don't even have to look at Firm 2. If one firm does not have a dominant strategy, then an equilibrium does not exist

# Iterative deletion of dominated strategies

Firm 2

| | $a_1$ / $a_2$ | Standard α | | Standard β | |
|---|---|---|---|---|---|
| Firm 1 | Standard α | 200 | 100 | 0 | 0 |
| | Standard β | 0 | 300 | 300 | 200 |

$$BR_1(a_2) = \begin{cases} \alpha & \text{if } a_2 = \alpha \\ \beta & \text{if } a_2 = \beta \end{cases} \qquad BR_2(a_1) = \begin{cases} \alpha & \text{if } a_1 = \alpha \\ \alpha & \text{if } a_1 = \beta \end{cases}$$

The above game does not have an equilibrium in dominant strategies. Does this mean that we cannot make any prediction? Still, we can if we delete Firm 2' dominated action (standard **β**)

In the "remaining" game, Firm 1 chooses Standard **α**, so (**α**,**α**) is our prediction

# Nash equilibrium

Firm 2

| $a_1$ / $a_2$ | Standard **α** | Standard **β** |
| --- | --- | --- |
| **Firm 1** Standard **α** | 200 100 | 0 0 |
| Standard **β** | 0 300 | 300 400 |

$$BR_1(a_2) = \begin{cases} \alpha & \text{if } a_2 = \alpha \\ \beta & \text{if } a_2 = \beta \end{cases}$$

$$BR_2(a_1) = \begin{cases} \alpha & \text{if } a_1 = \alpha \\ \beta & \text{if } a_1 = \beta \end{cases}$$

A Nash equilibrium (NE) is an outcome that "lies" on the BR function of each player

This game has 2 NE outcomes: (**α**, **α**) and (**β**, **β**) [compatibility]

Intuitively, a player cannot increase his payoff by deviating given that no one else deviates

# A Nash equilibrium does not always exist (standardization game)

Firm 2

Firm 1

| $a_1$ / $a_2$ | Standard α | | Standard β | |
|---|---|---|---|---|
| Standard α | 200 | 100 | 0 | 200 |
| Standard β | 0 | 300 | 300 | 200 |

$$BR_1(a_2) = \begin{cases} \alpha & \text{if } a_2 = \alpha \\ \beta & \text{if } a_2 = \beta \end{cases} \qquad BR_2(a_1) = \begin{cases} \beta & \text{if } a_1 = \alpha \\ \alpha & \text{if } a_1 = \beta \end{cases}$$

A Nash equilibrium (NE) does not exist

Intuitively, firm 1 seeks standard compatibility whereas firm 2 wants to operate on a different standard

# A Nash equilibrium does not always exist (penalty kicks in soccer)

Goalie

Kicker

| $a_1$ / $a_2$ | Dive Left | Dive Right |
|---|---|---|
| Kick Left | 0 , 1 | 1 , 0 |
| Kick Right | 1 , 0 | 0 , 1 |

Using BR functions show that a Nash equilibrium does not exist

Players may use mixed strategies: Kickers will kick left with probability ½. Goalie will dive left with probability ½.

See also: Chiappori, Levitt, & Groseclose. "Testing Mixed-Strategy Equilibria When Players Are Heterogeneous: The Case of Penalty Kicks in Soccer." American Economic Review, 2002.

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# The Prisoner's' Dilemma: Example

Firm 2

| | $a_1$ / $a_2$ | Low Price (L) | | High Price (H) | |
|---|---|---|---|---|---|
| Firm 1 | Low Price (L) | 100 | 100 | 300 | 0 |
| | High Price (H) | 0 | 300 | 200 | 200 |

(L,L) is an equilibrium in dominant strategies (hence, also NE)

However, colluding on (H,H) would yield higher profit to <u>each</u> player! That is, (H,H) Pareto dominates (L,L).

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# The Prisoner's' Dilemma: General formulation

Player 2

| $a_1$ / $a_2$ | Cooperate | Defect |
|---|---|---|
| Cooperate | a a | c b |
| Defect | b c | d d |

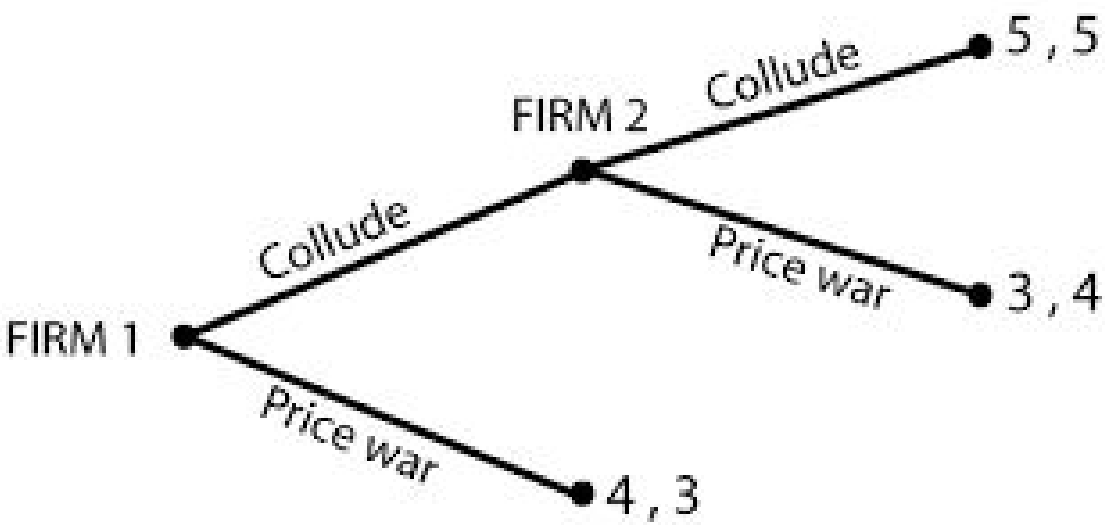
Player 1

Let b > a > d > c, so that (Defect,Defect) is a NE, but both players could be made better off under (Cooperate, Cooperate)


Golden balls video

# Multistage games: Two types

1. Simultaneous moves: The same game (say, the single-stage prisoner's dilemma) is repeated more than once:
   a. Finitely-many times, or
   b. infinitely-many times
2. Sequential moves: Players take turns after observing the rival's play: Examples: Chess, Checkers




# Multistage (sequential moves) game: The Ultimatum Game: Playing for real !!!


There are 6 candy bars on the table.
Two-stage (two-player) game. Instructions:

1. Player 1: Divide the bars: Make an offer of X to player 2 (6 − X for yourself), where $X \in \{0, 1, 2, 3, 4, 5, 6\}$

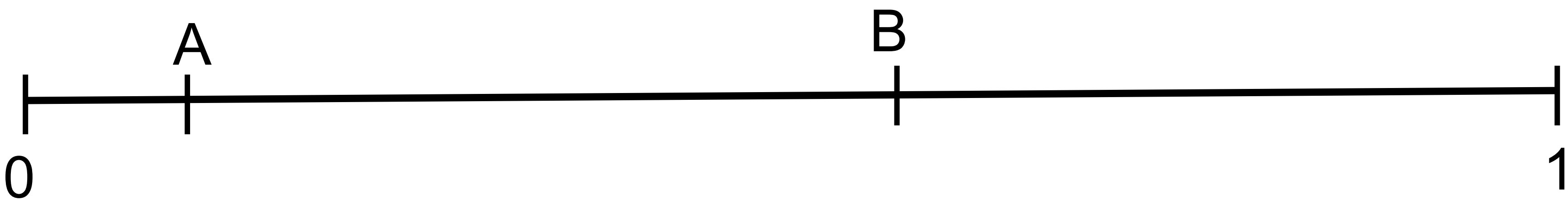
2. Player 2: Choose between: Agree or Disagree

Agree → $\pi_1 = 6 - X$ and $\pi_2 = X$

Disagree → $\pi_1 = \pi_2 = 0$

# Location models of the linear city

- We simplify today's discussion by assuming that prices are fixed at P (not a price game, as in Hotelling (1929))
- 2 shops (A & B) located somewhere on the interval [0, 1]
- Continuum of buyers residing uniformly on [0, 1]
- Given equal prices, consumers shop at the store (rest




Class discussion: Given fixed (say, regulated ) prices ($P), where would shop A and shop B choose to located in a simultaneous-moves game?

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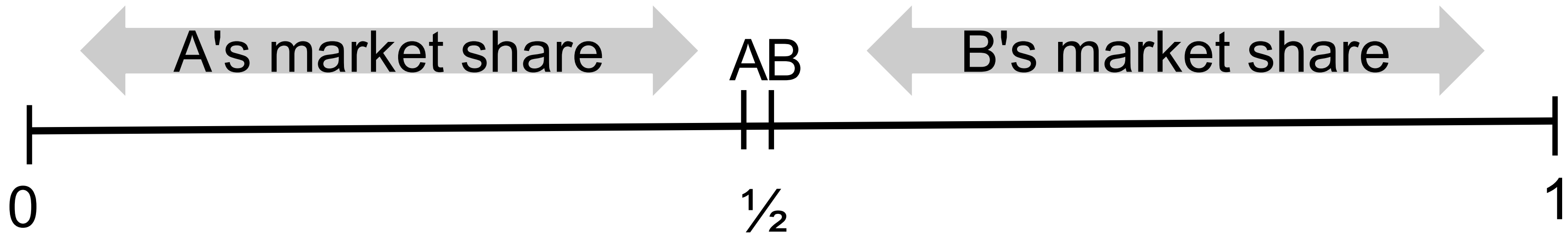
# Location models of the linear city

Answer: If both stores are 'forced' to charge the same price, P, then they will located as close as possible to each other at the city's midpoint




Very important remark: If stores can set their own prices, stores will find it profitable to move away from each other to create product differentiation!